Rayleigh-Taylor instability in confined porous media: pore-scale simulations and experiments

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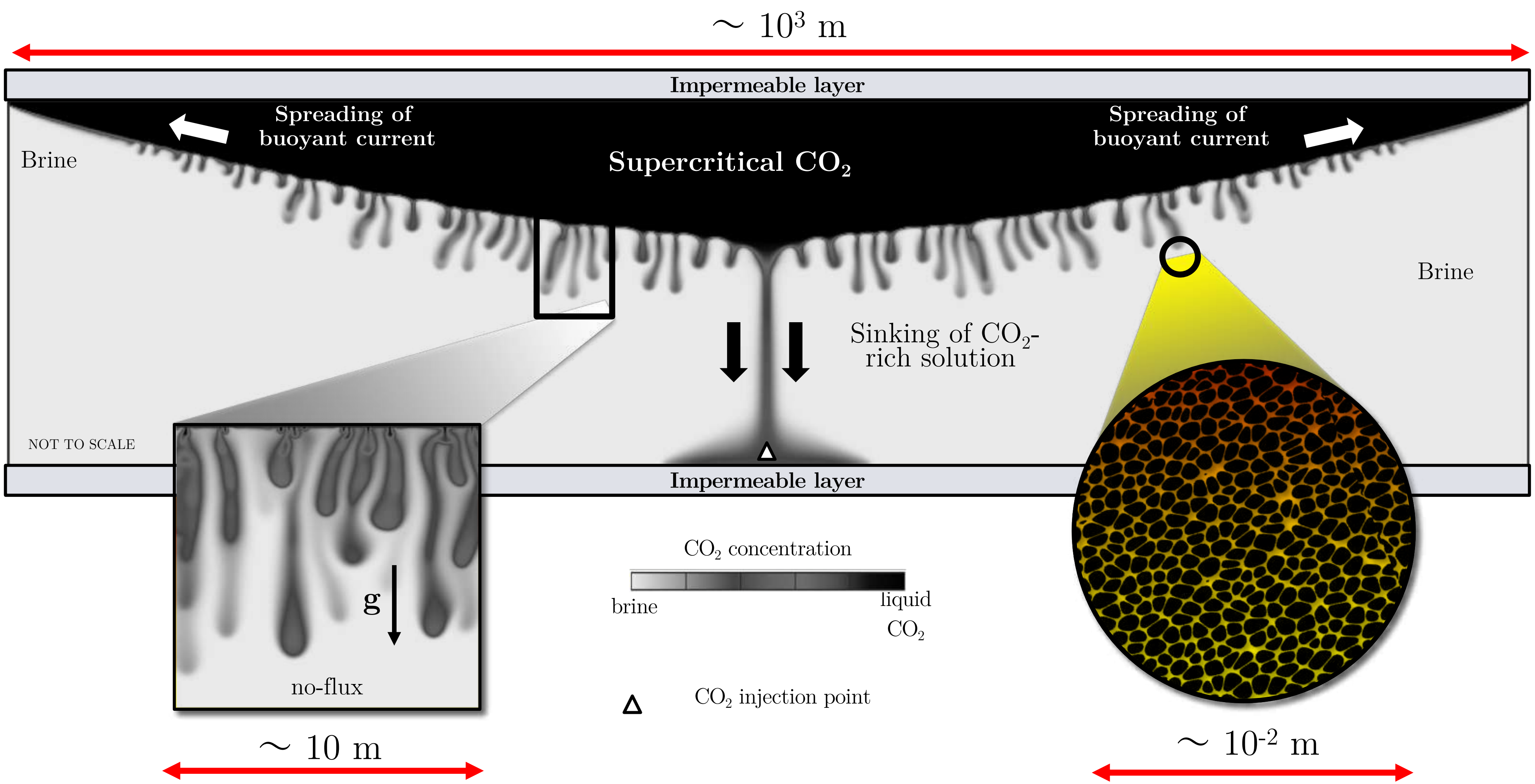
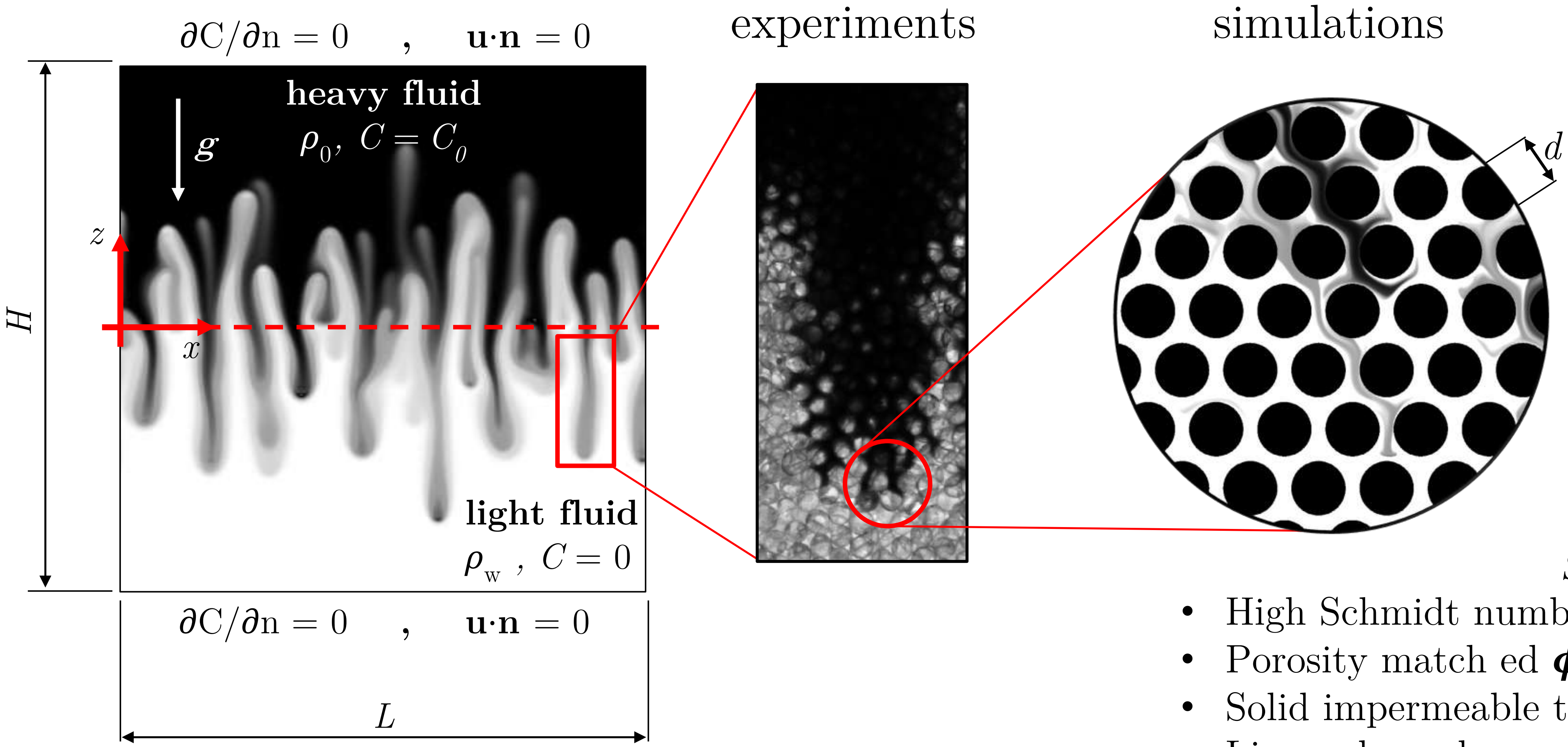
Motivation



De Paoli, *Phys. Fluids.* (2021)

Flow configuration

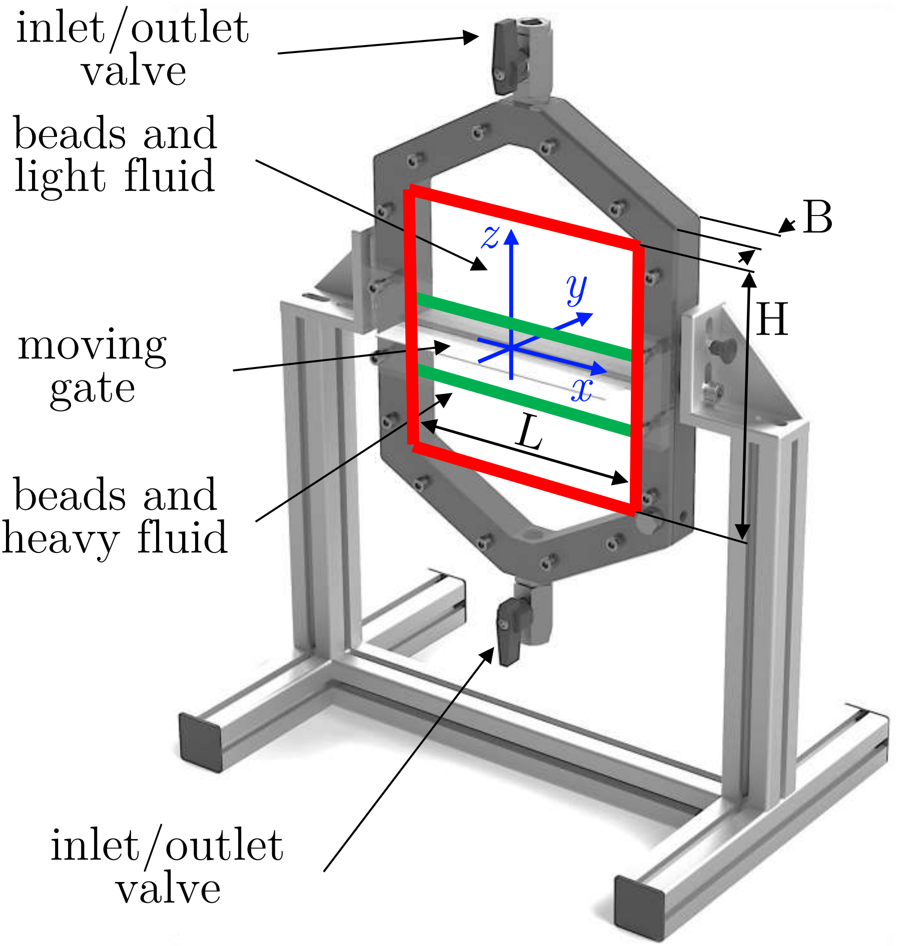


$$Sc = \frac{\mu}{\rho_0 D}$$

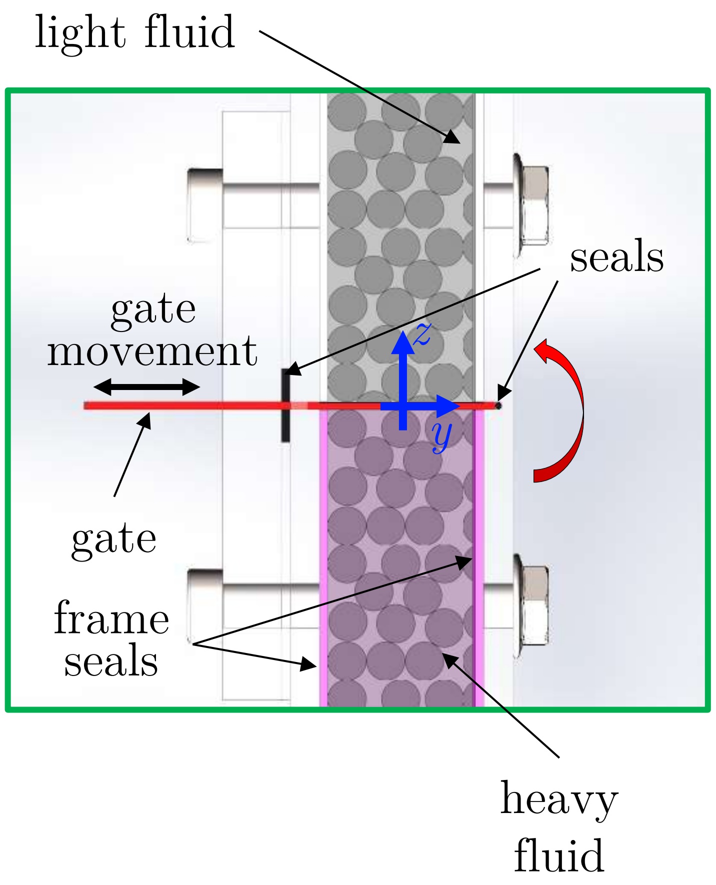
- High Schmidt number
- Porosity match ed $\phi = 0.37$
- Solid impermeable to solute
- Linear dependency $\rho(C)$

Experimental setup

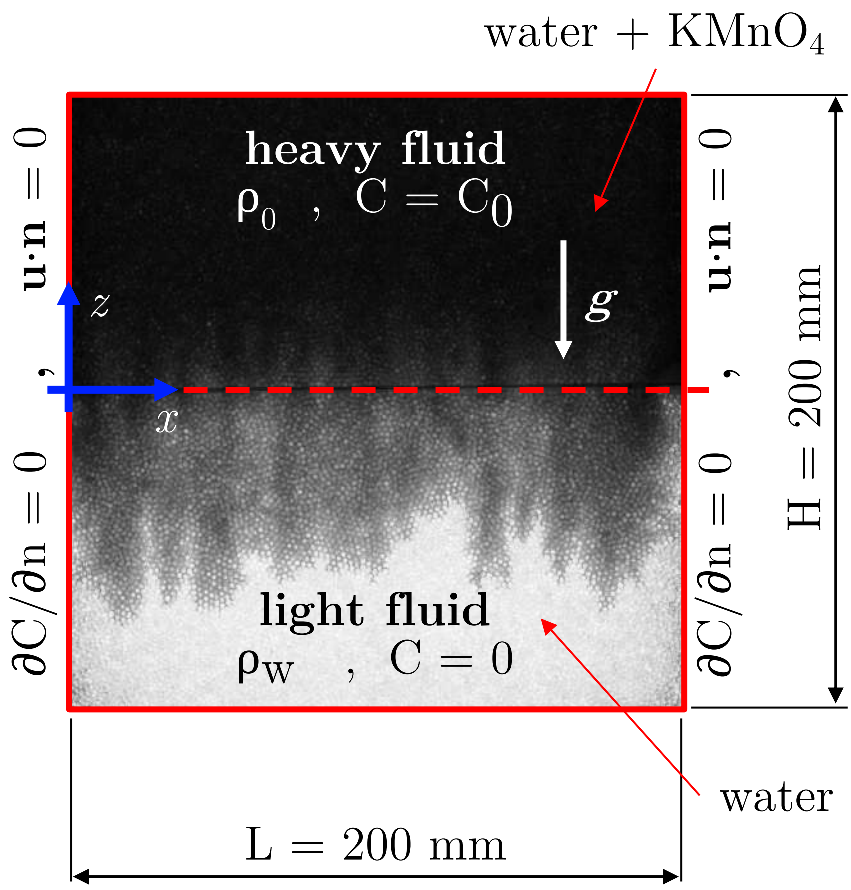
(a) Hele-Shaw cell



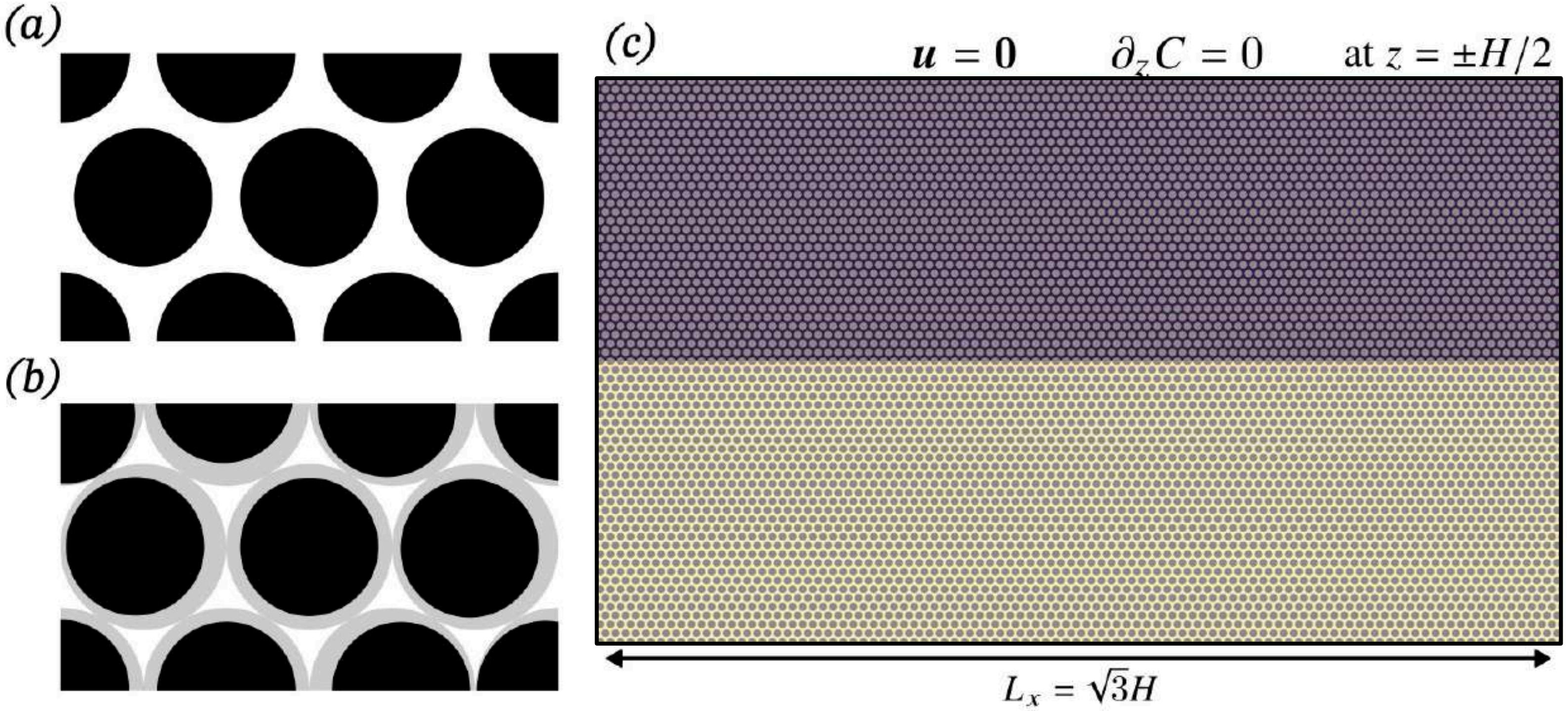
(b) gate (side view)



(c) measurement region



Numerical method



$$\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\boldsymbol{u} = -\rho_0^{-1}\boldsymbol{\nabla} p + \nu \nabla^2 \boldsymbol{u} - g\beta C \hat{\boldsymbol{z}},$$

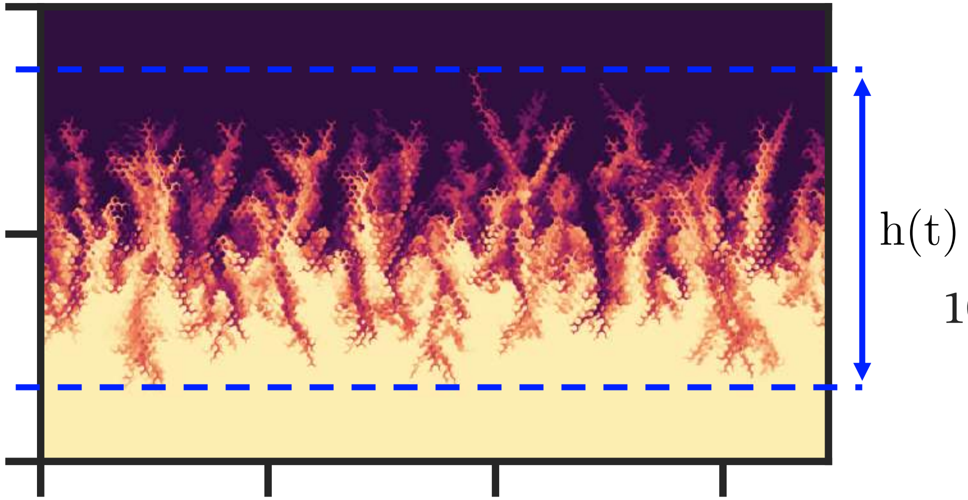
$$\partial_t C + (\boldsymbol{u} \cdot \boldsymbol{\nabla})C = D\nabla^2 C,$$

$$\rho = \rho_0 \left[1 + \frac{\Delta\rho}{\rho_0 C_0}(C - C_0) \right]$$

Finite difference
(AFiD, open source)
+
Immersed Boundaries
Method

Resolution:
- velocity: $\geq$ 32 points per diameter
- conc.: $\geq$ 128 points per diameter

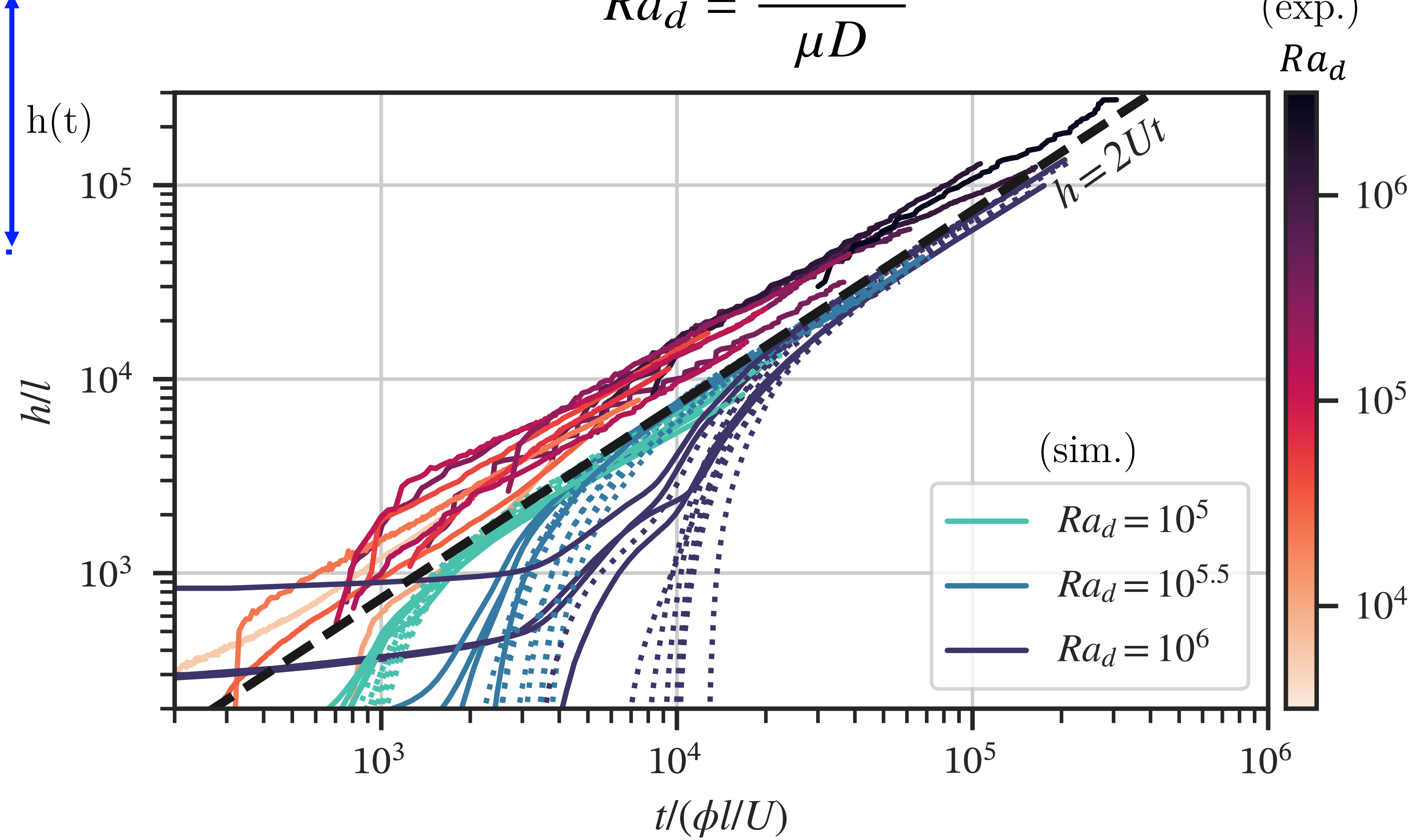
Mixing length



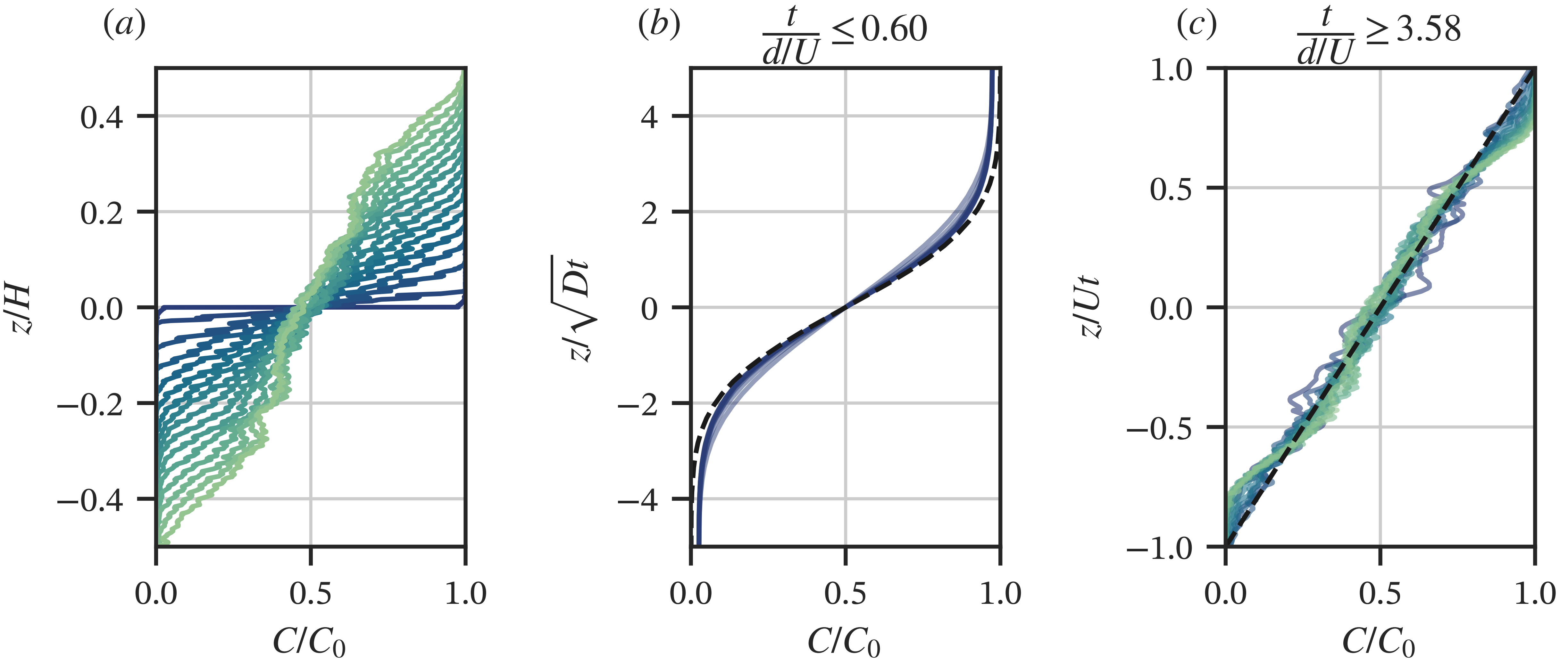
$$U = \frac{g\Delta\rho k}{\mu}$$

$$\ell = \frac{\phi D}{U}$$

$$Ra_d = \frac{g\Delta\rho d^3}{\mu D}$$



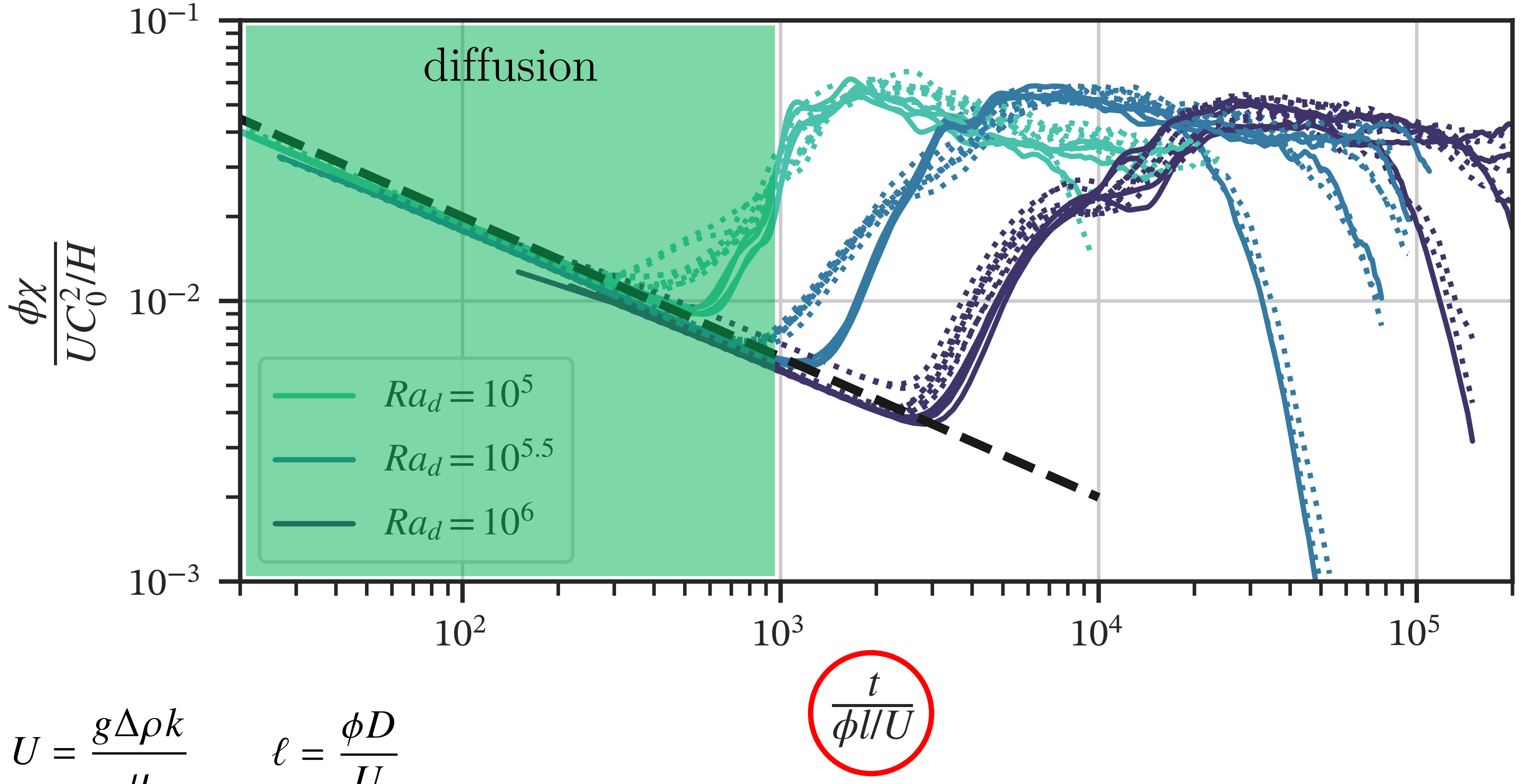
Concentration profiles



Modelling scalar dissipation

$$\chi = D\langle|\nabla C|^2\rangle_f = \frac{D}{V_f}\int_{V_f}|\nabla C|^2 \, dV$$

Can we model this mixing/dissolution process?



$U = \frac{g\Delta\rho k}{\mu}$ $\quad \ell = \frac{\phi D}{U}$

Diffusion:

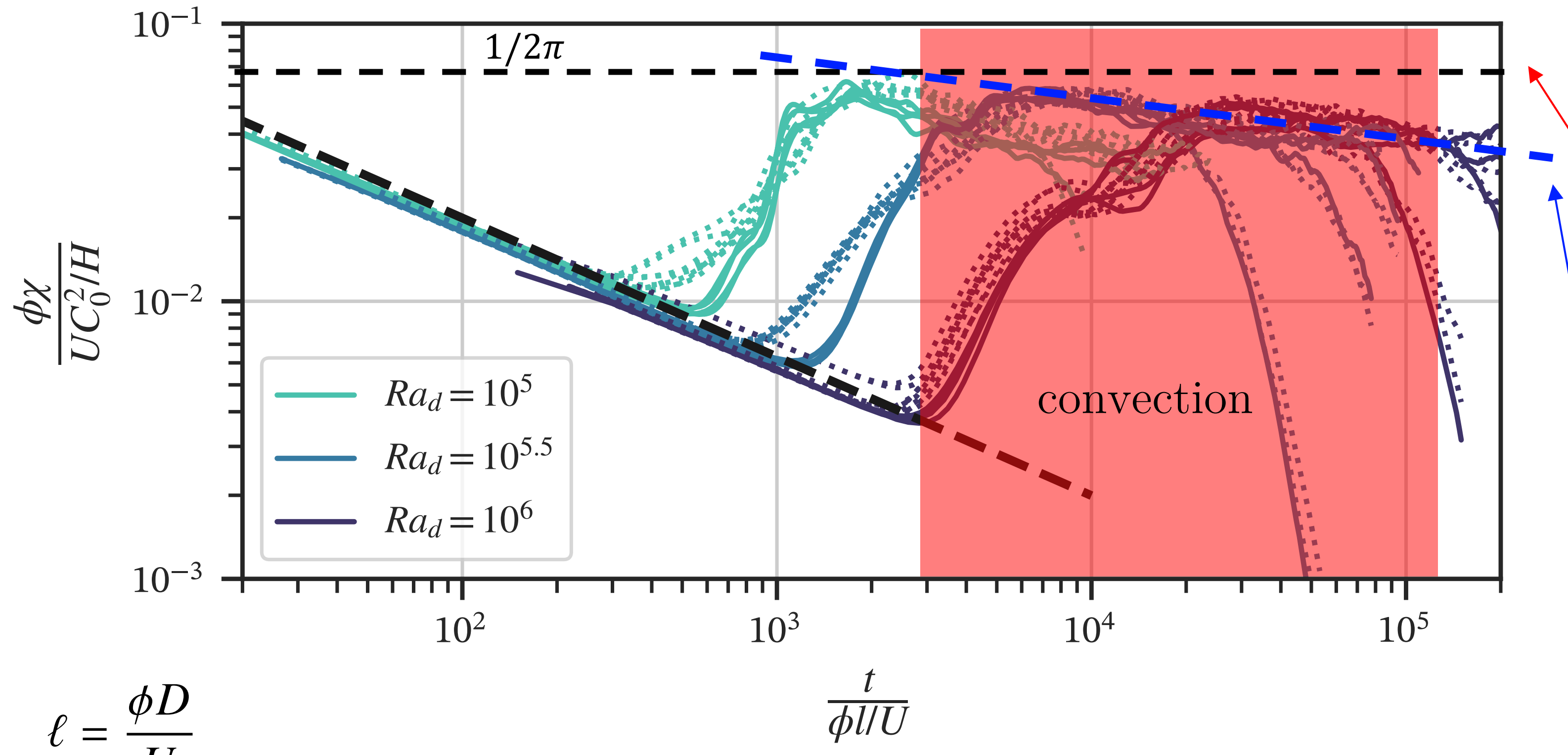
$$C = C_0 + \frac{\Delta C}{2}\mathrm{erf}\left(\frac{z}{\sqrt{2\kappa t}}\right)$$

$$\partial_z C = \frac{\Delta C}{2\sqrt{\pi\kappa t}}\exp\left(-\frac{z^2}{2\kappa t}\right)$$

$$\chi = \kappa\langle|\nabla C|^2\rangle = \frac{\kappa}{H}\int_{-\infty}^{\infty}|\partial_z C|^2 \mathrm{d}z = \sqrt{\frac{\kappa}{8\pi t}}\frac{(\Delta C)^2}{H}$$

Modelling scalar dissipation

$$\chi = D\langle|\nabla C|^2\rangle_f = \frac{D}{V_f}\int_{V_f}|\nabla C|^2\, dV$$



$$\ell = \frac{\phi D}{U}$$

Convection

$$\chi = \kappa\langle|\nabla C|^2\rangle = \kappa\frac{L_m}{H}\langle|\nabla C|^2\rangle_{ML},$$

$$|\nabla C| \approx \frac{\Delta C}{2\sqrt{\pi\kappa t}}.$$

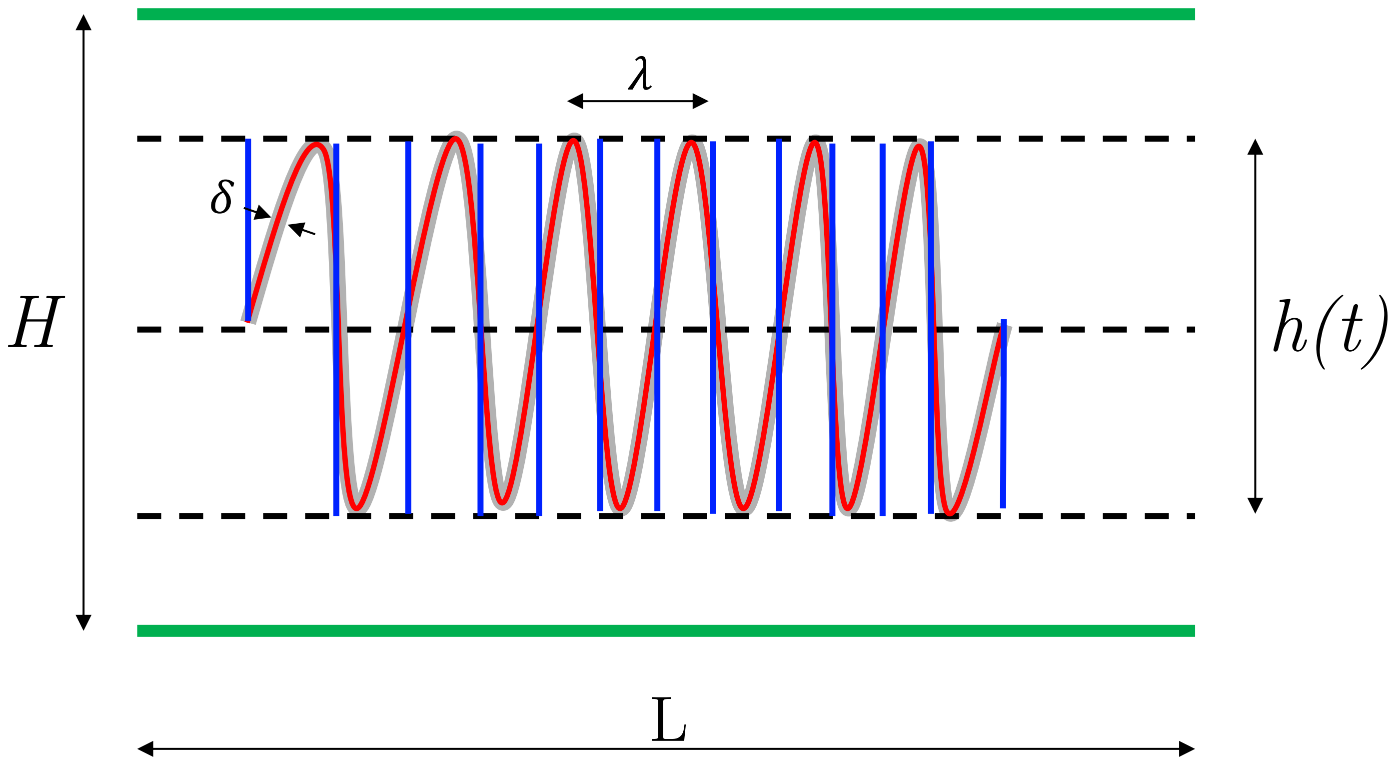
$$L_m \approx 2Ut,$$

$$\chi \approx \kappa\frac{2Ut}{H}\frac{(\Delta C)^2}{4\pi\kappa t} = \frac{1}{2\pi}\frac{U_d(\Delta C)^2}{H}.$$

$$\boxed{\frac{\phi\chi}{UC_0^2/H} = 1/(2\pi)}$$

$1/(2\pi)$ is the maximum value ofmean dissipation. Measurements indicate that χ decreases with time

Modelling scalar dissipation



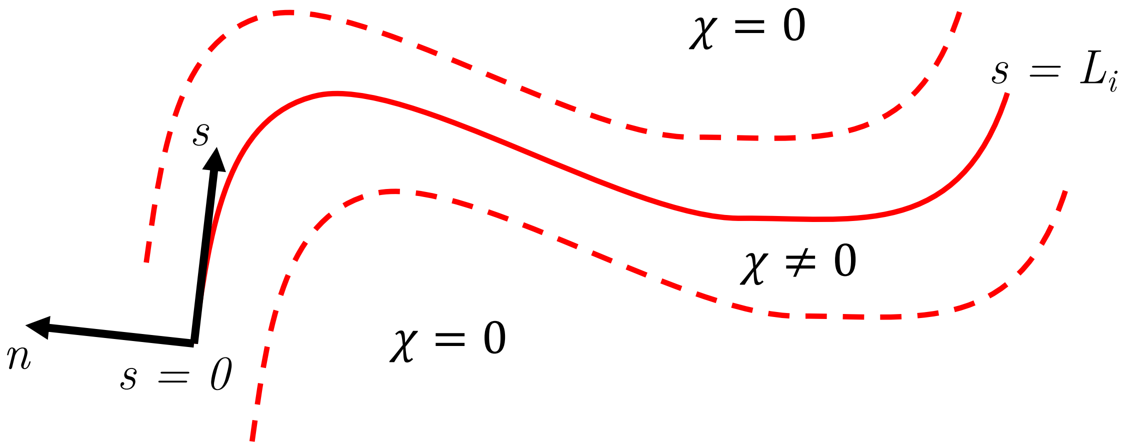
$$\chi = D\langle|\nabla C|^2\rangle = \frac{D L_i}{HL}\int_{-\delta/2}^{+\delta/2} |\partial_n C|^2 \, dn$$

Assume:

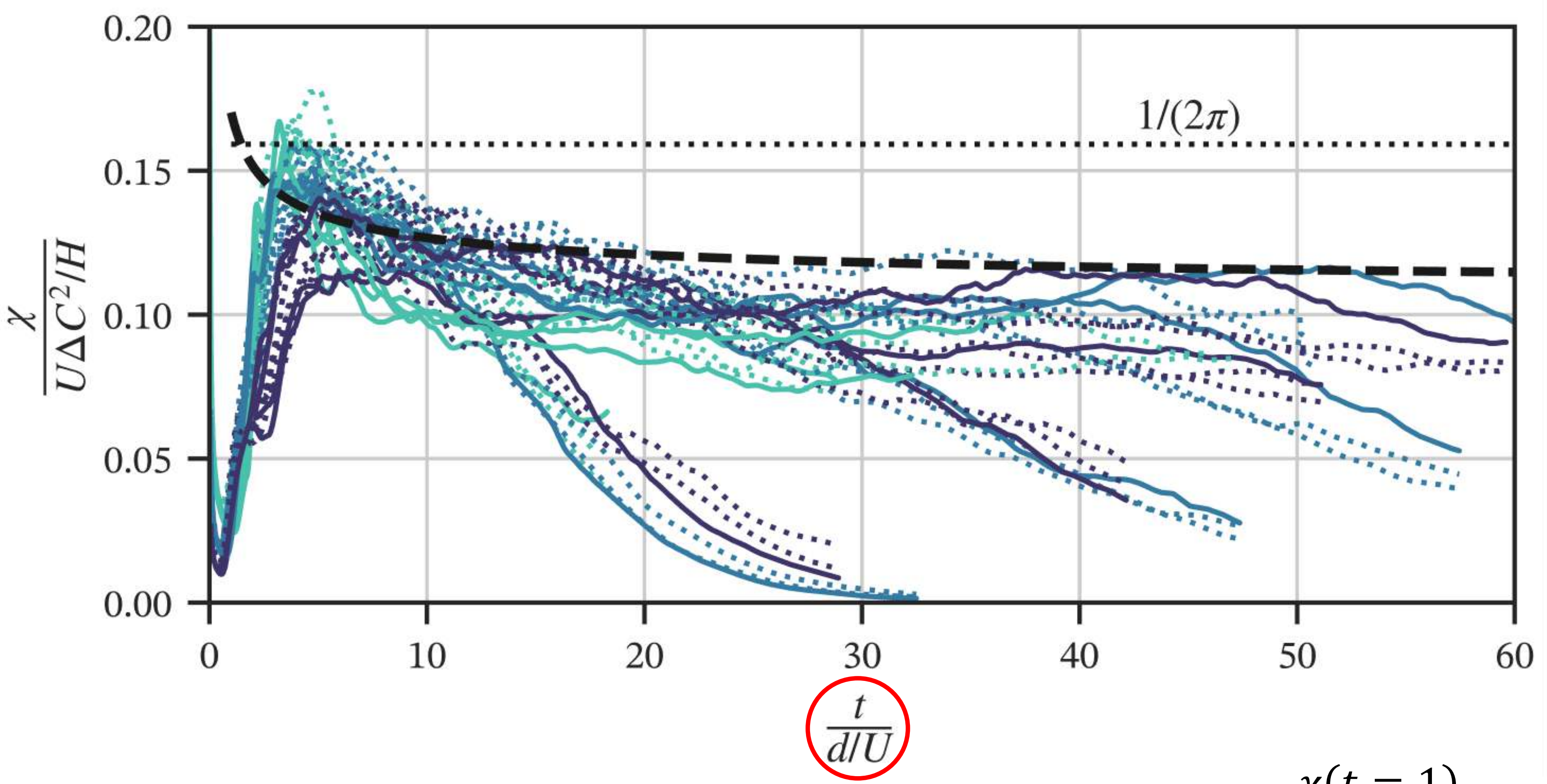
1) Interface grows as:

$$L_i = L + 2\,N_{finger}\,h = L + 2\frac{L}{\lambda}h$$

2) Gradient across the interface evolves according to the diffusive solution

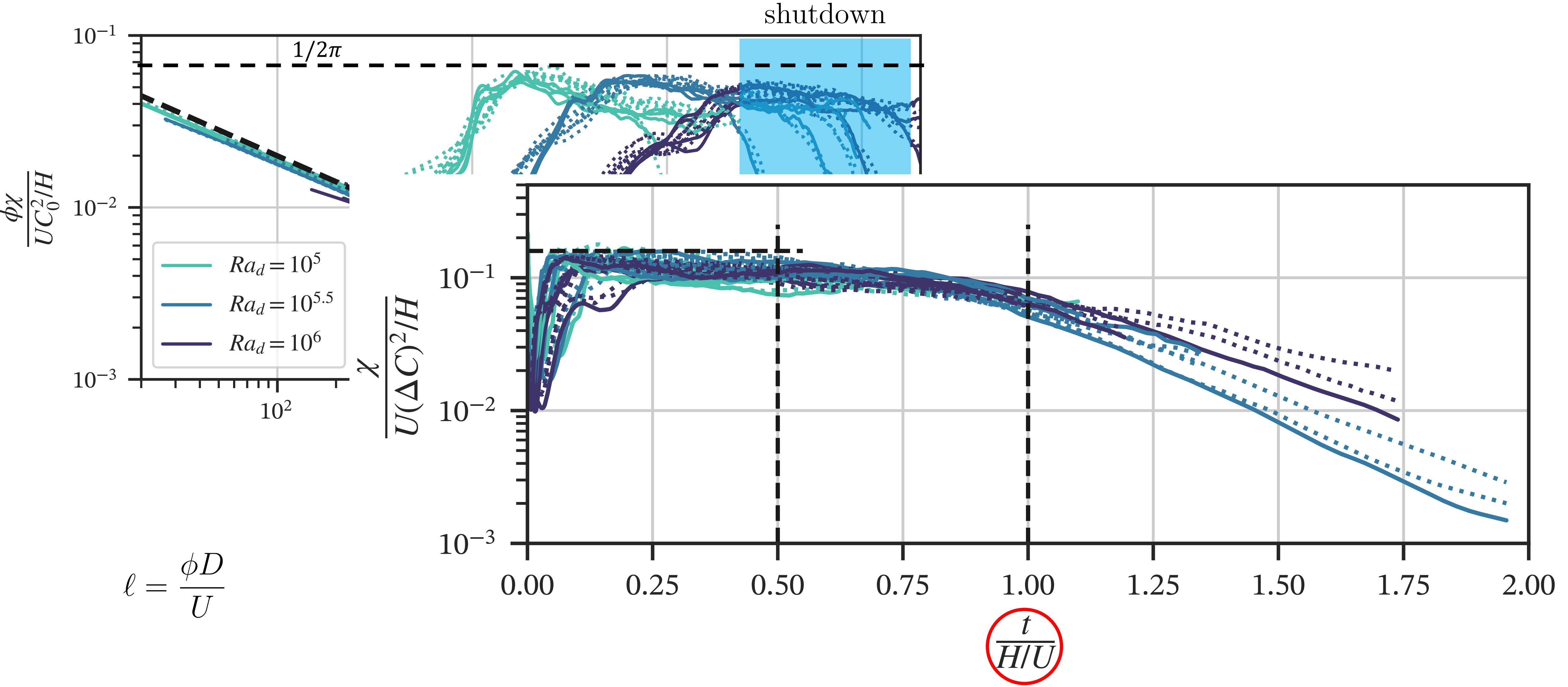


Modelling scalar dissipation



$$\frac{\chi(t=1)}{(\Delta C)^2 U/H} = \frac{\beta}{\alpha\pi}\left(1+\frac{\alpha}{4}\right) \approx \frac{1}{1.92\pi} \approx \frac{1}{2\pi}$$

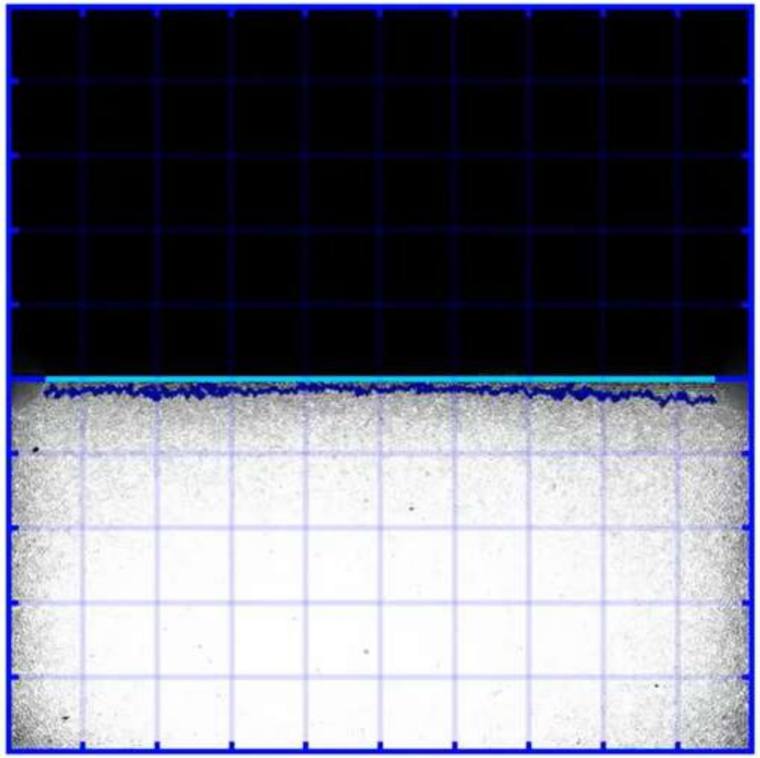
Modelling scalar dissipation



$$\ell = \frac{\phi D}{U}$$

Conclusions

- Multiple **length scales** are relevant to **different phases** of the process
- We explain theoretically the scaling laws observed
- We plan to performed simulations in three-dimensional domains and **Darcy simulations** with **dispersion**



PREPRINT

arxiv.org/abs/2310.04068

European Commission

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MAX-PLANCK-GESELLSCHAFT

FWF Der Wissenschaftsfonds.

PRACE

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Thank you for your attention! Questions?

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High-resolution images, movies and slides are available upon request to m.depaoli@utwente.nl