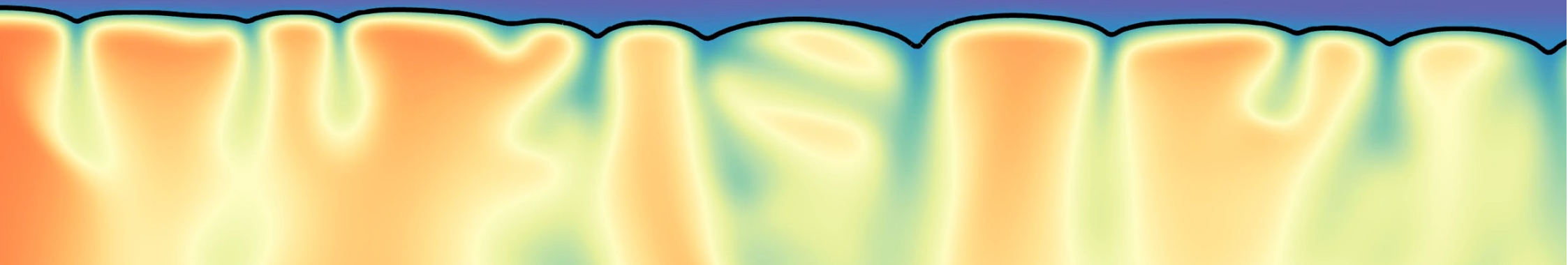


Mixing of complex fluids in confined porous media



Marco De Paoli¹ & Sergio Pirozzoli²

¹TU Wien, Vienna, Austria

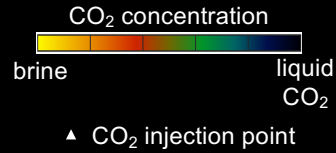
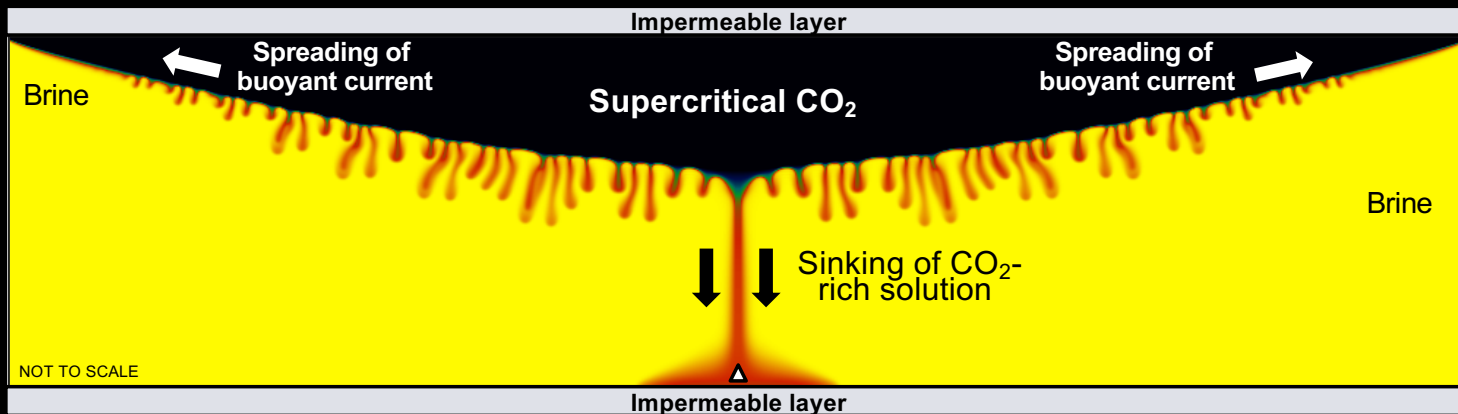
²Sapienza University, Rome, Italy

marco.de.paoli@tuwien.ac.at

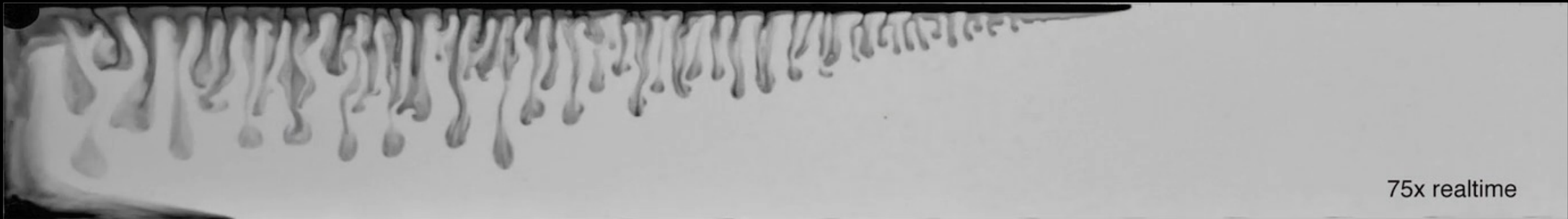


De Paoli, *Phys. Fluids* (2021)

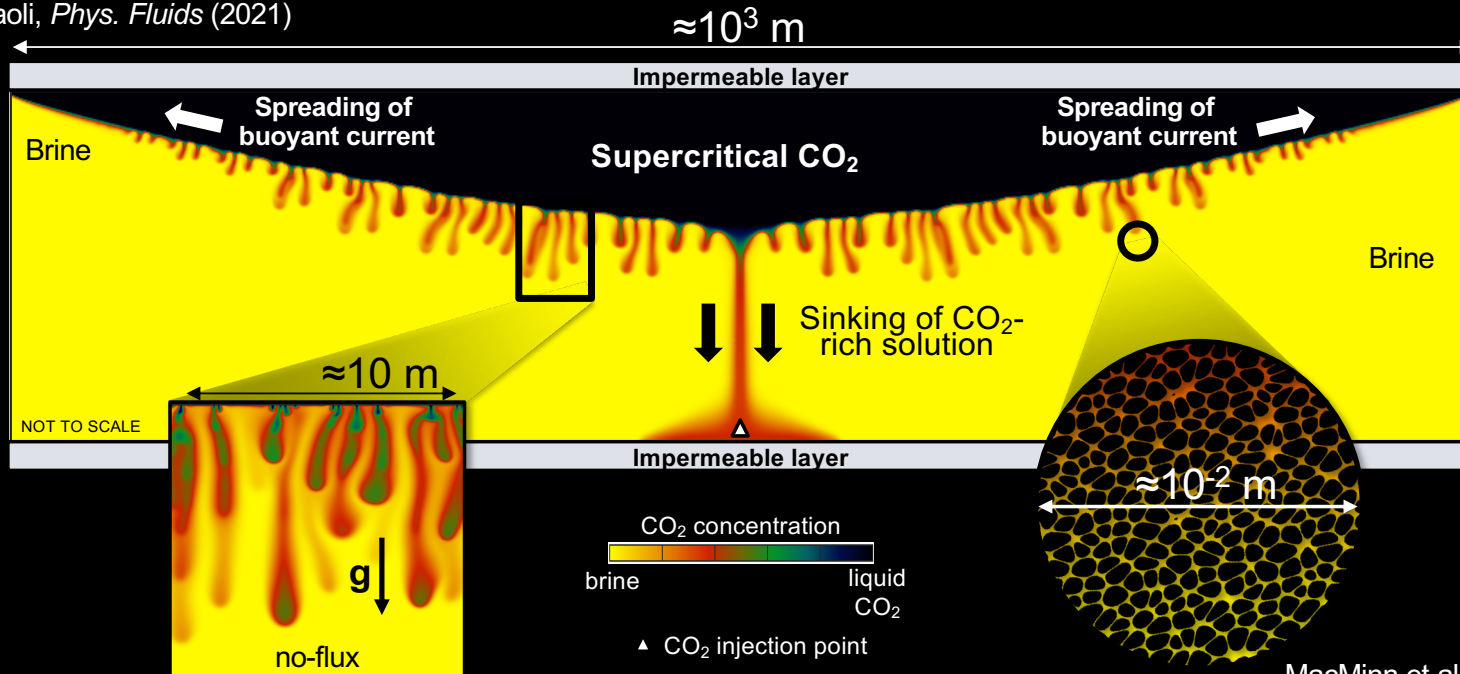
$\approx 10^3$ m



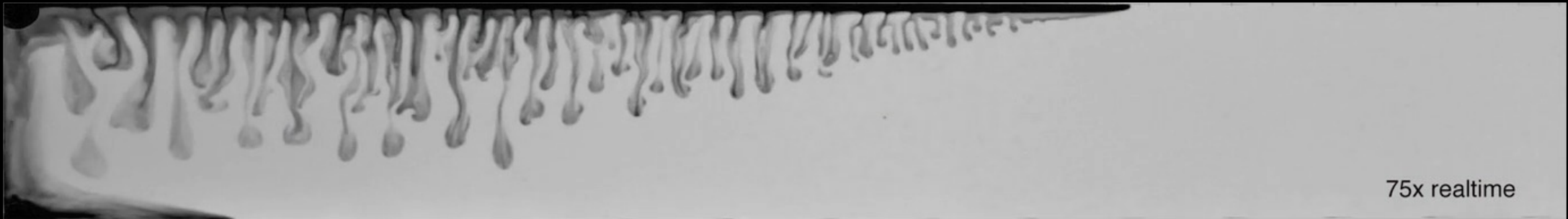
MacMinn et al., *Geophys. Res. Lett.* (2013)



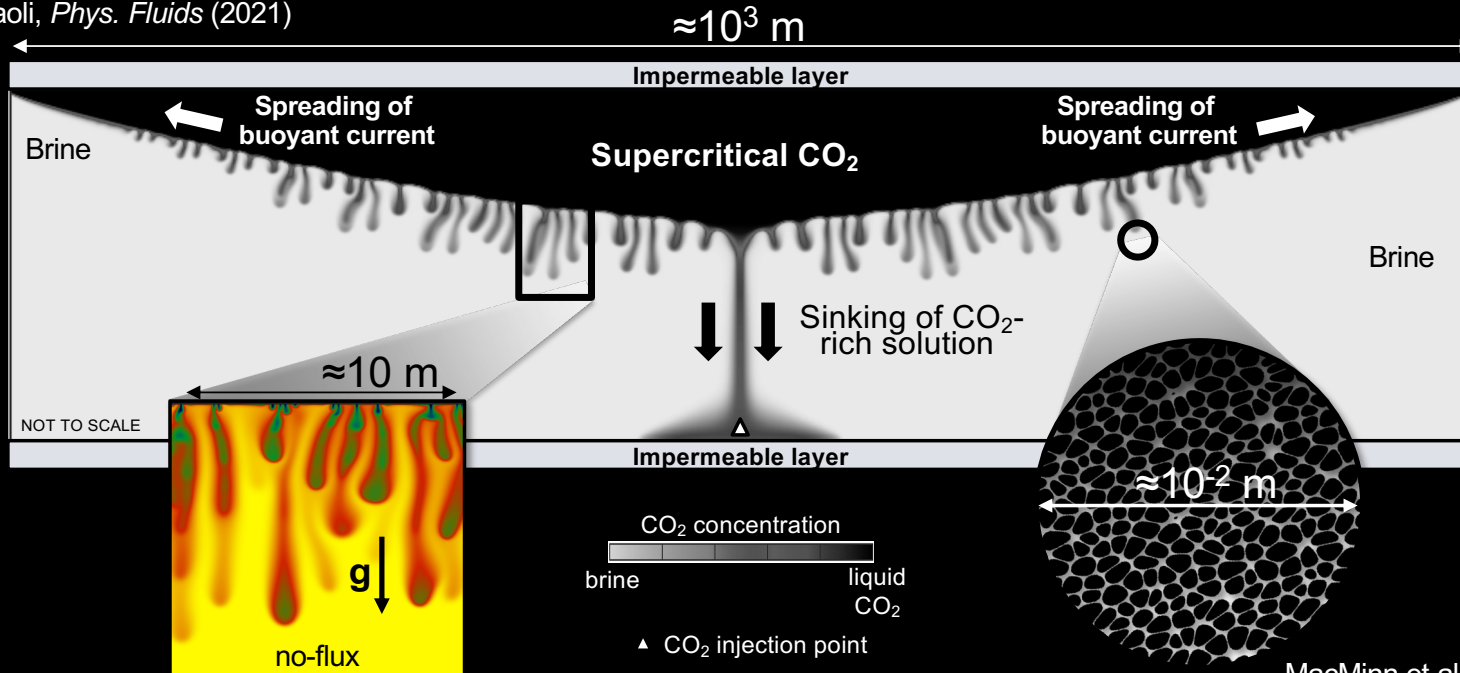
De Paoli, *Phys. Fluids* (2021)



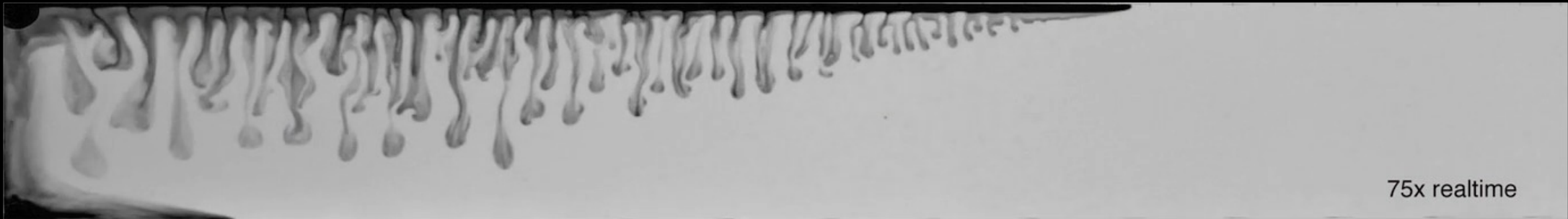
MacMinn et al., *Geophys. Res. Lett.* (2013)



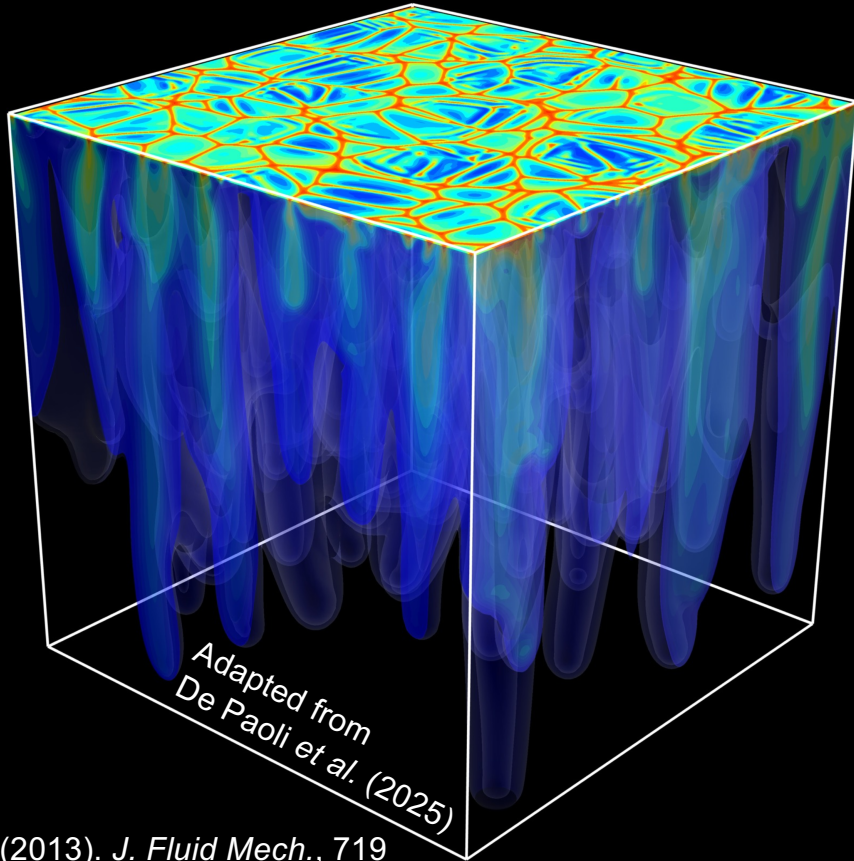
De Paoli, *Phys. Fluids* (2021)



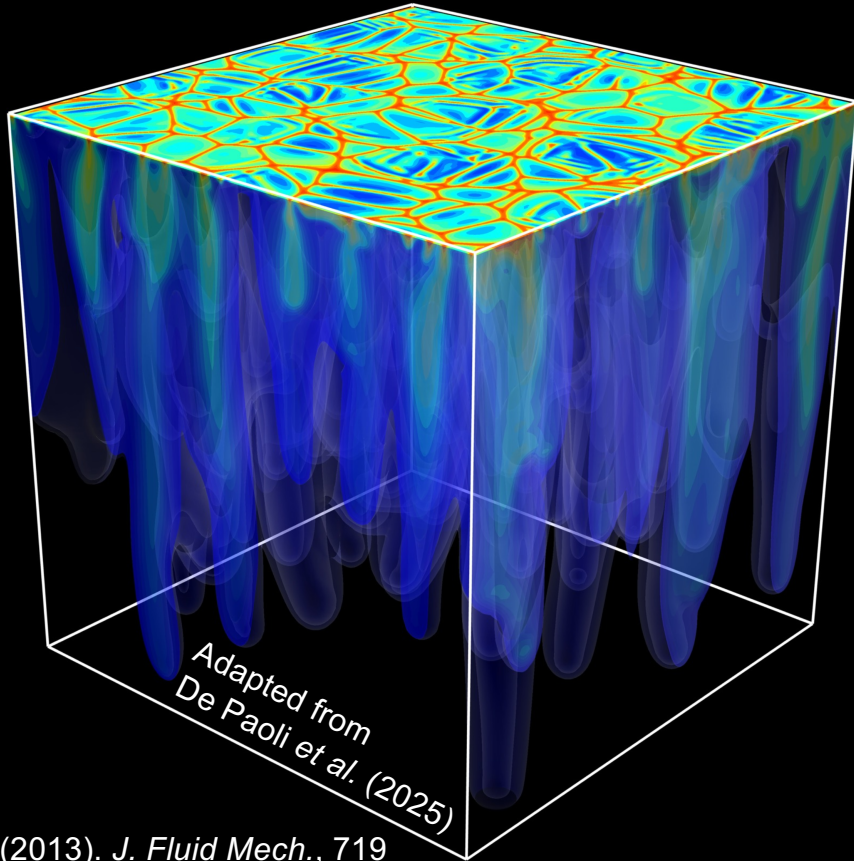
MacMinn et al., *Geophys. Res. Lett.* (2013)



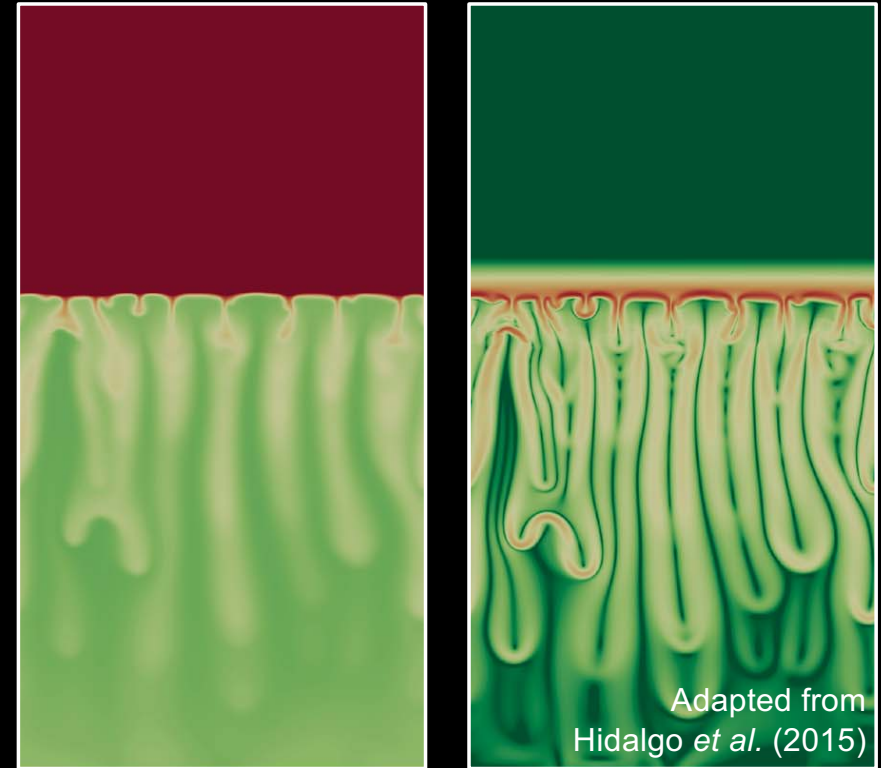
Fixed interface

Adapted from
De Paoli et al. (2025)Hewitt et al. (2013). *J. Fluid Mech.*, 719Slim, A. (2014). *J. Fluid Mech.*, 741De Paoli et al. (2025). *Geophys. Res. Lett.*, 52 (7)

Fixed interface



Free interface



Hewitt et al. (2013). *J. Fluid Mech.*, 719

Slim, A. (2014). *J. Fluid Mech.*, 741

De Paoli et al. (2025). *Geophys. Res. Lett.*, 52 (7)

Hewitt et al. (2013). *J. Fluid Mech.*, 719

Hidalgo et al. (2015). *Geophys. Res. Lett.*, 42 (15)

Imuetinyan et al. (2026). *The Eur. Phys. J. E*, 49 (20)

What is the role of boundary conditions?

What is the role of boundary conditions?

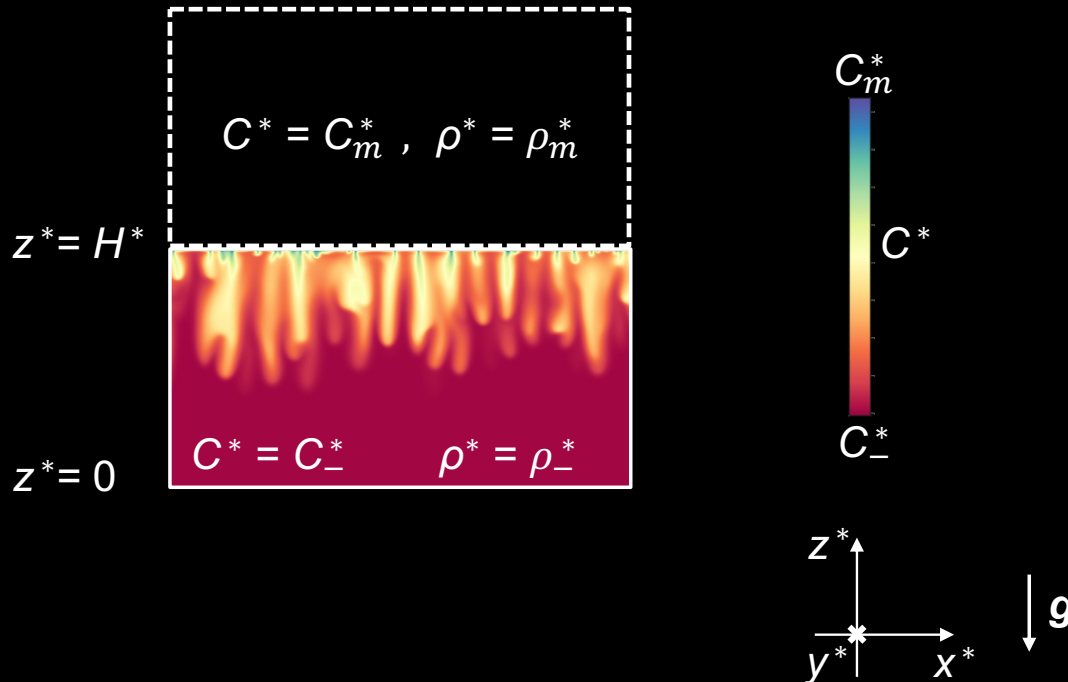
What is the role of fluid model?

What is the role of boundary conditions?

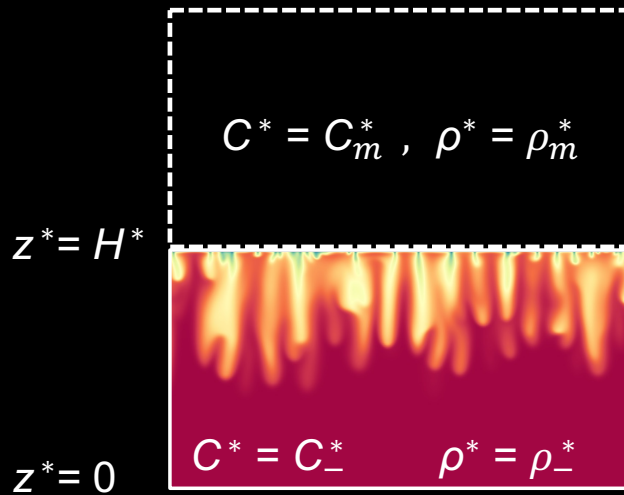
What is the role of fluid model?

What is the role of dimensionality?

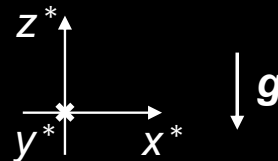
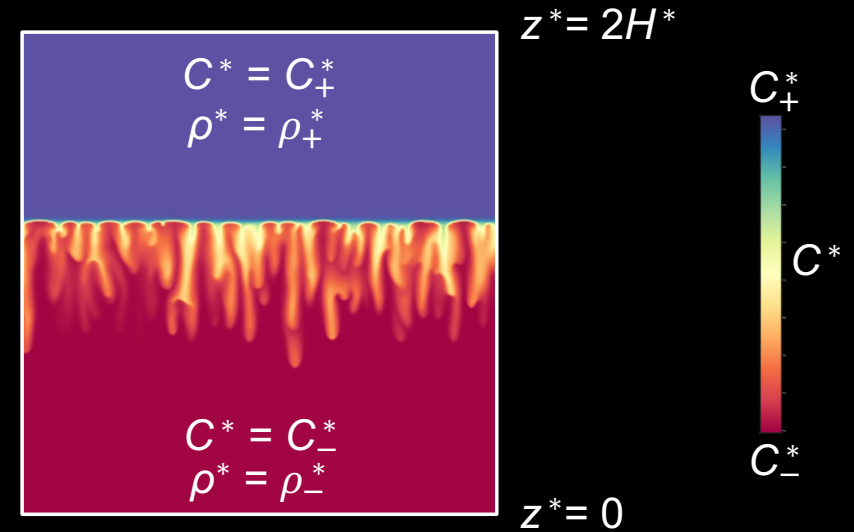
(a) fixed interface



(a) fixed interface



(b) free interface



Flow

$$\nabla \cdot \mathbf{u} = 0$$

$$\mathbf{u} = -(\nabla p + \rho \mathbf{k})$$

$$\frac{\partial C}{\partial t} + \mathbf{u} \cdot \nabla C = \frac{1}{Ra_0} \nabla^2 C$$

Flow

$$\nabla \cdot \mathbf{u} = 0$$

$$\mathbf{u} = -(\nabla p + \rho \mathbf{k})$$

$$\frac{\partial C}{\partial t} + \mathbf{u} \cdot \nabla C = \frac{1}{Ra_0} \nabla^2 C$$

Fluid

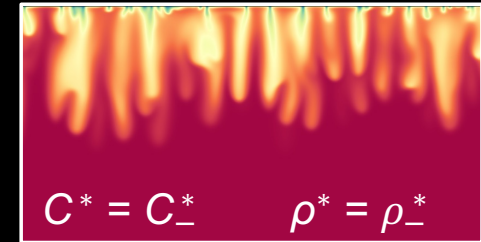
Fixed interface

$$\rho = 1 + C, \quad -1 \leq C \leq 0$$

$$\rho = 1 - C^2, \quad -1 \leq C \leq 0$$

$$C^* = C_m^*, \quad \rho^* = \rho_m^*$$

$z^* = H^*$



$$C^* = C_-^*, \quad \rho^* = \rho_-^*$$

$z^* = 0$

Flow

$$\nabla \cdot \mathbf{u} = 0$$

$$\mathbf{u} = -(\nabla p + \rho \mathbf{k})$$

$$\frac{\partial C}{\partial t} + \mathbf{u} \cdot \nabla C = \frac{1}{Ra_0} \nabla^2 C$$

Fluid

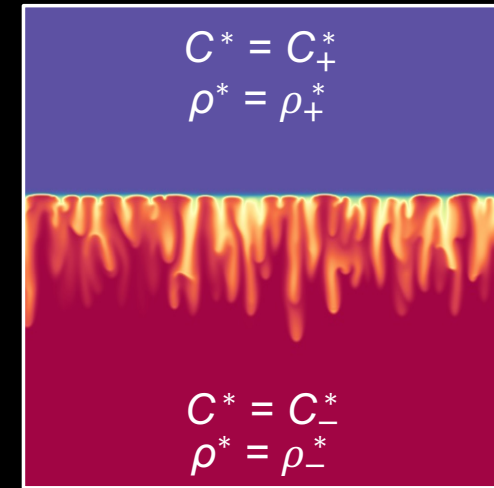
Fixed interface

$$\rho = 1 + C, \quad -1 \leq C \leq 0$$

$$\rho = 1 - C^2, \quad -1 \leq C \leq 0$$

Free interface

$$\rho = \begin{cases} 1 - C^2, & -1 \leq C \leq 0 \\ 1 - (1 + \beta) \left(1 - \frac{1}{\alpha}\right)^2 C^2, & 0 < C \leq C_+ \end{cases}$$



$z^* = 2H^*$

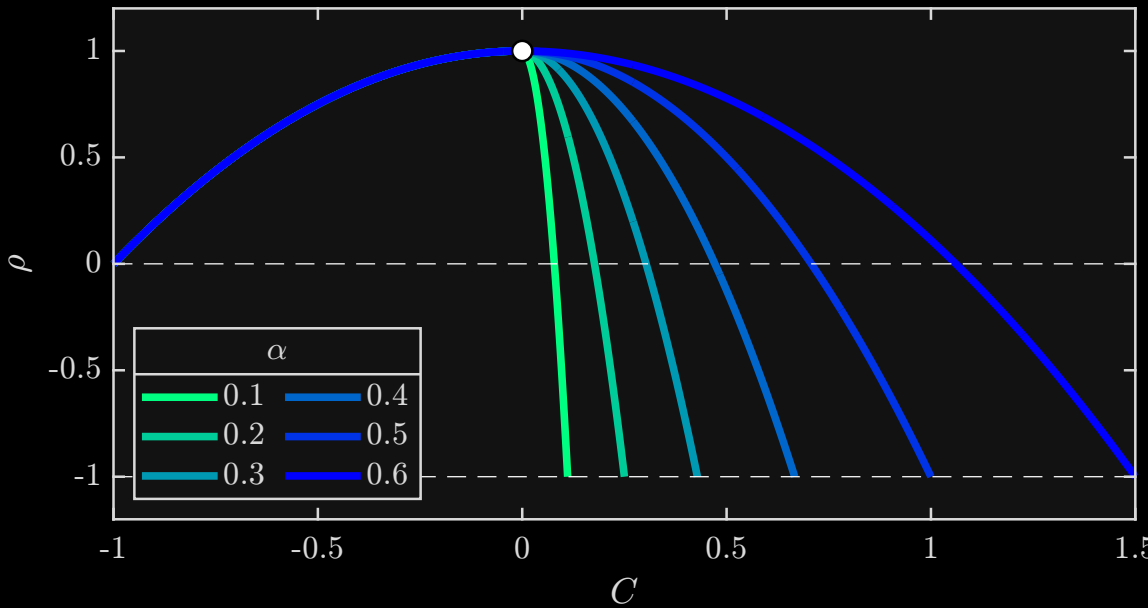
$z^* = 0$

$$0 < C \leq C_+ = \frac{\alpha}{1 - \alpha}$$

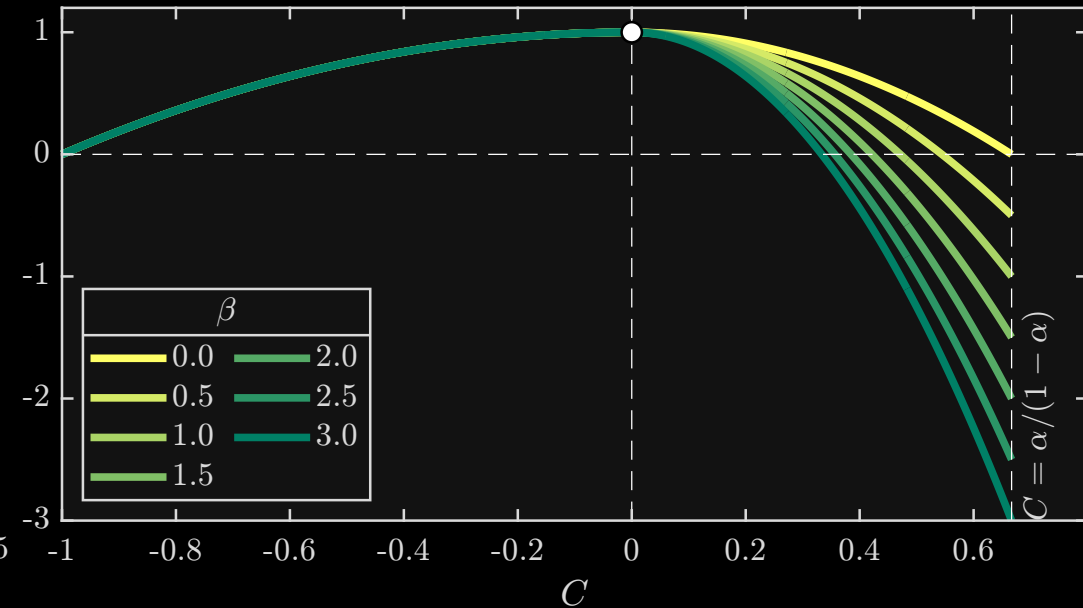
$$Ra_0 = \frac{\mathcal{U}^* H^*}{\phi D}, \quad \alpha = \frac{C_+^* - C_m^*}{C_+^* - C_-^*}, \quad \beta = \frac{\rho_-^* - \rho_+^*}{\rho_m^* - \rho_-^*}$$

De Paoli & Pirozzoli (2026), arXiv:2604.23199

Effect of α

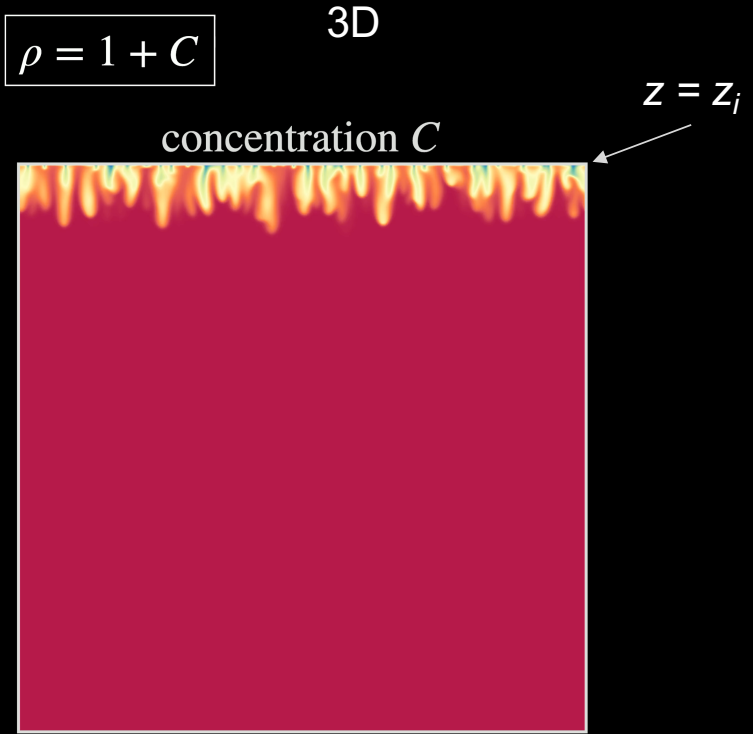


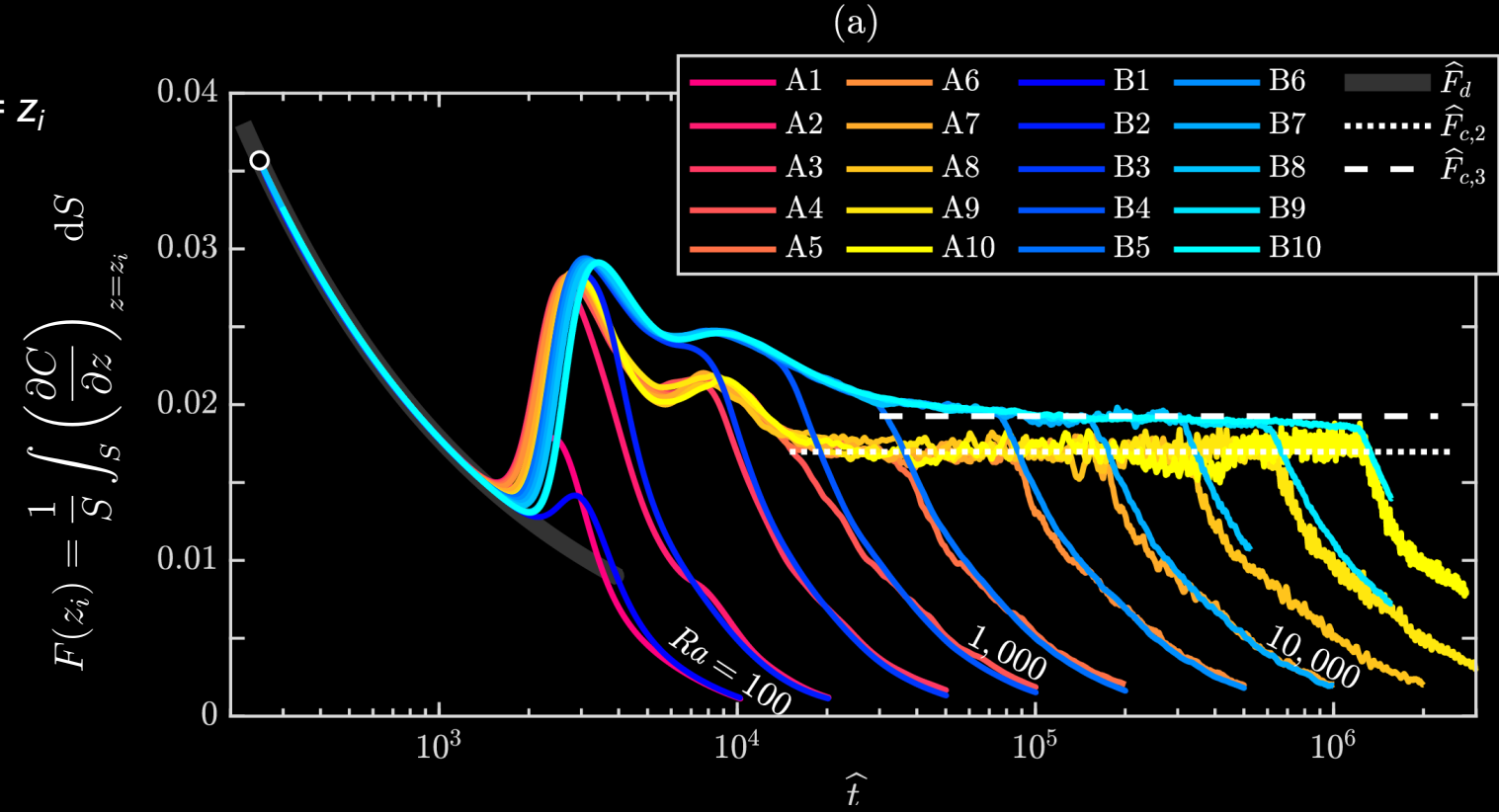
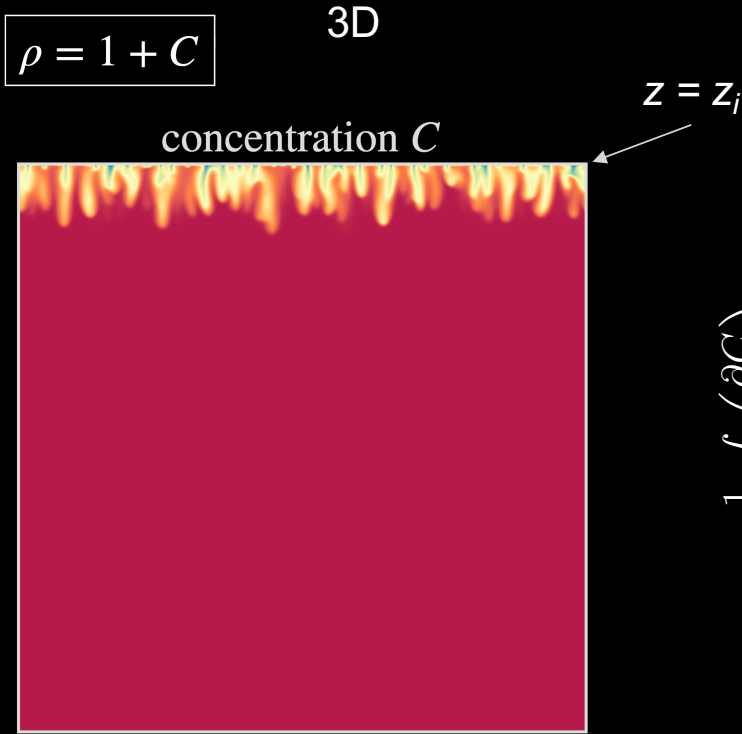
Effect of β



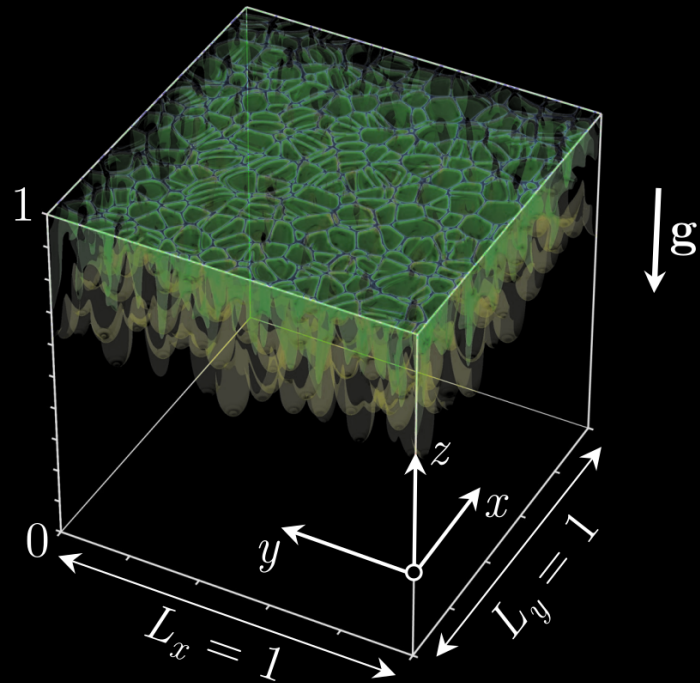
$$Ra_0 = \frac{\mathcal{U}^* H^*}{\phi D} \quad , \quad \alpha = \frac{C_+^* - C_m^*}{C_+^* - C_-^*} \quad , \quad \beta = \frac{\rho_-^* - \rho_+^*}{\rho_m^* - \rho_-^*}$$

In this work
 $Ra_0 = 10^4$

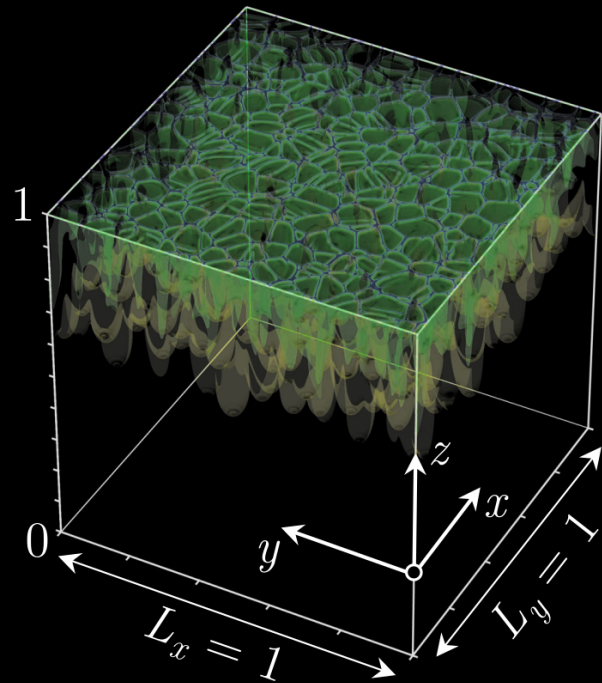




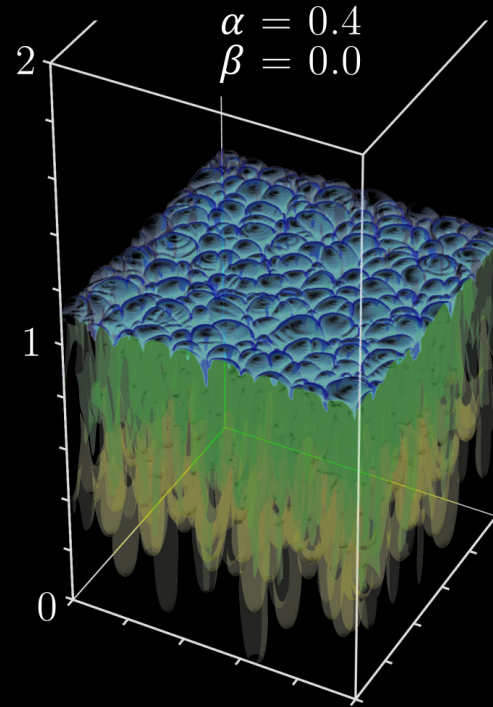
fixed interface
(linear)



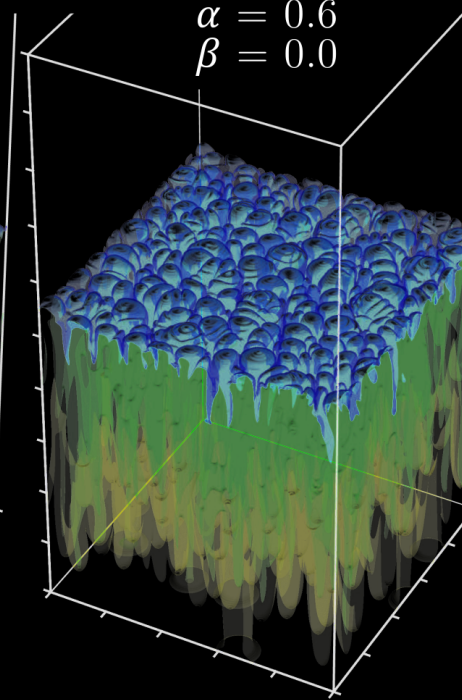
fixed interface
(linear)



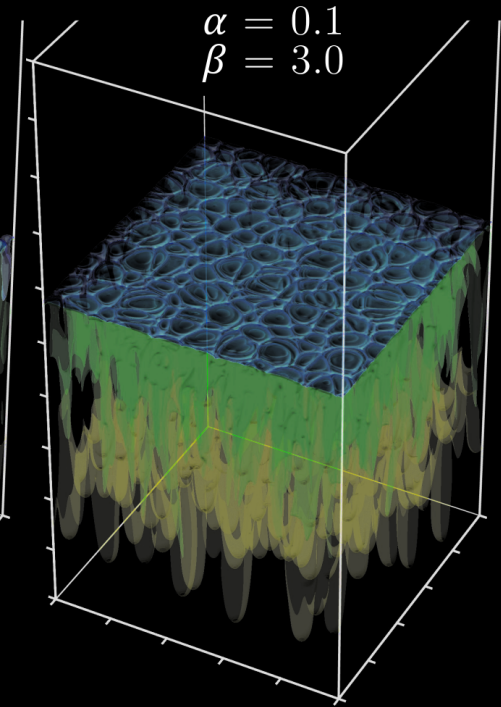
free interface



deformed interface



deformed interface

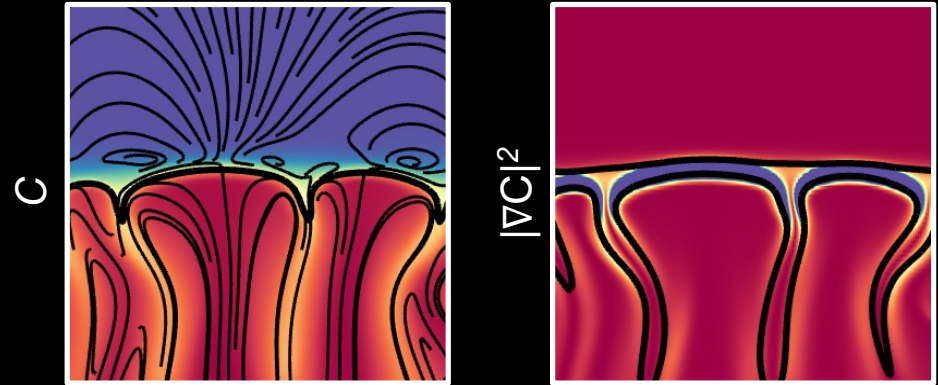


flat interface

Mean scalar dissipation

$$\chi = \langle |\nabla C|^2 \rangle$$

with $\langle \quad \rangle$ = volume average

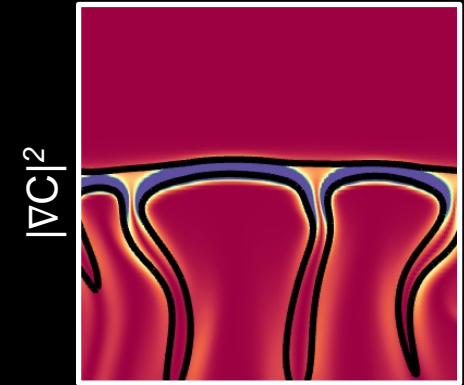
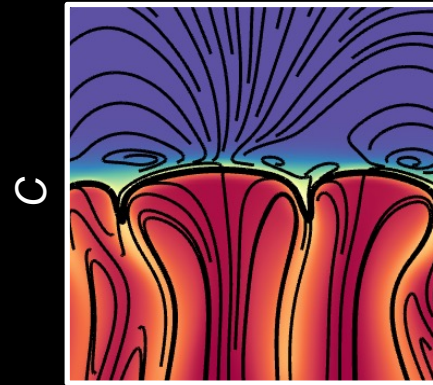


- Jha, B., Cueto-Felgueroso, L., & Juanes, R. (2011). *Phys. Rev. E*, 84 (6), 066312
Hidalgo, J., Fe, J., Cueto-Felgueroso, L., & Juanes, R. (2012). *Phys. Rev. Lett.*, 109 (26), 264503.
Hidalgo, J., Dentz, M., Cabeza, Y., & Carrera, J. (2015). *Geophys. Res. Lett.*, 42 (15), 6357–6364.

Mean scalar dissipation

$$\chi = \langle |\nabla C|^2 \rangle$$

with $\langle \quad \rangle =$ volume average



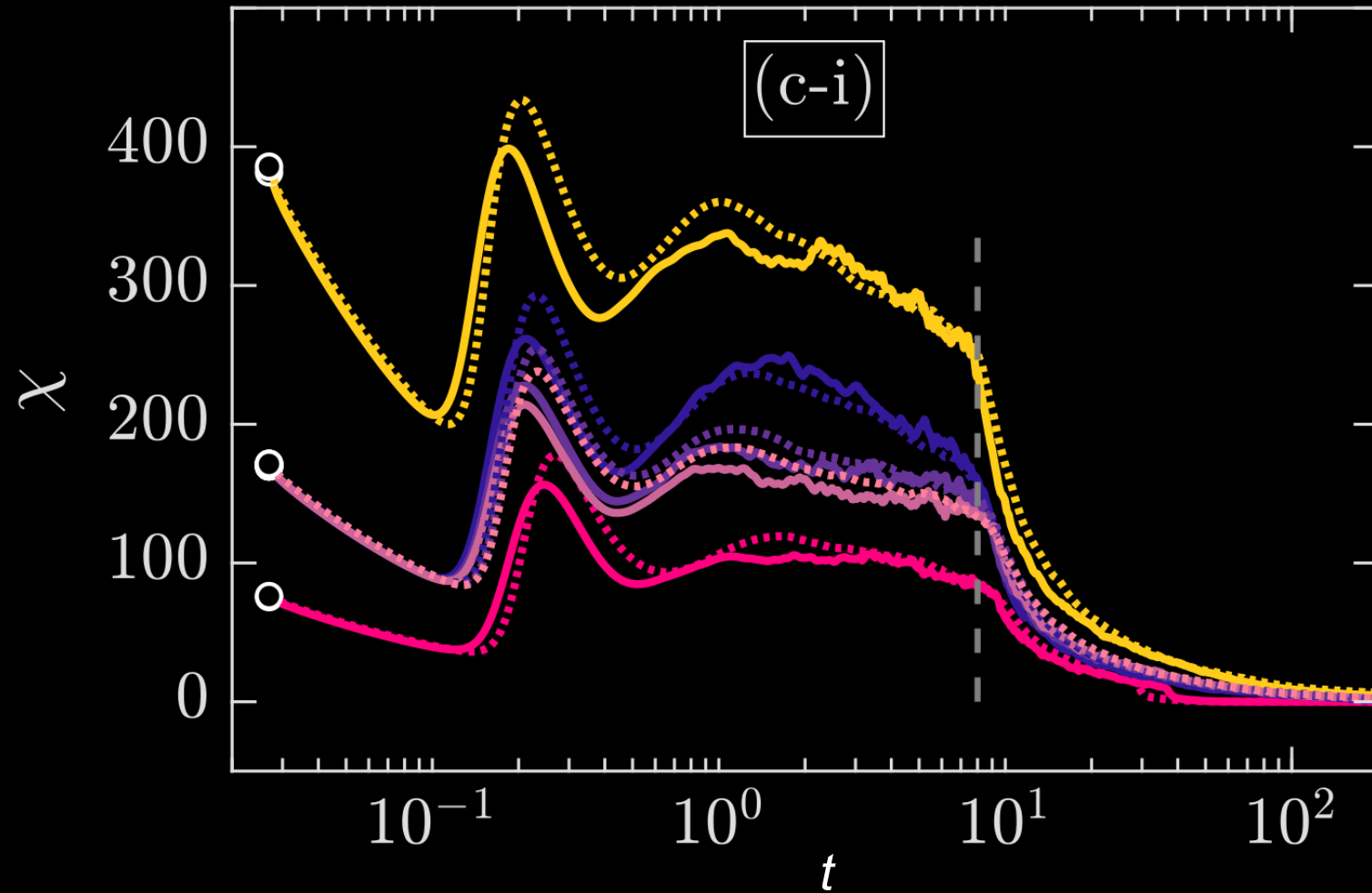
Degree of mixing

$$M(t) = 1 - \frac{\sigma^2(t)}{\sigma_{\max}^2} = \frac{8(1 - \alpha)^2}{Ra} \int_0^t \chi dt$$

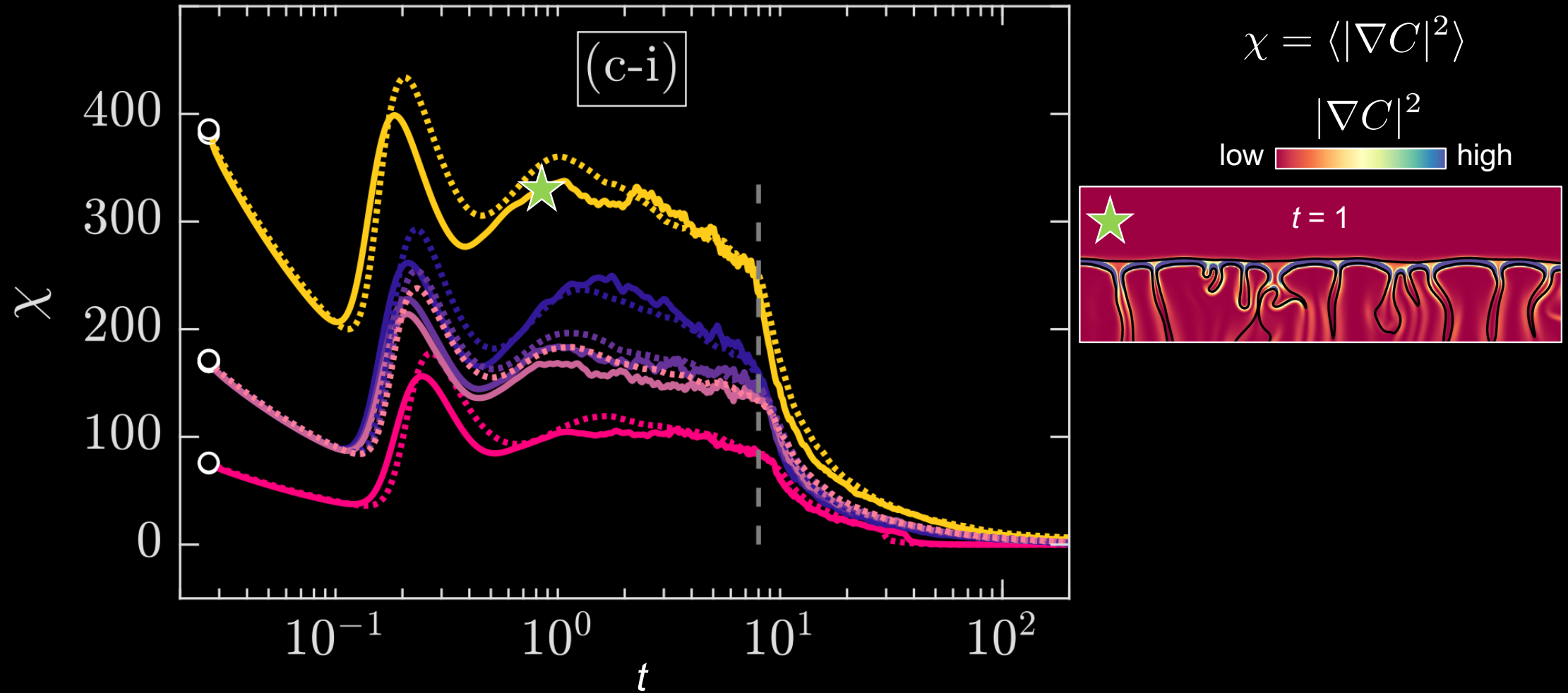
$$\sigma^2 = \langle C^2 \rangle - \langle C \rangle^2$$

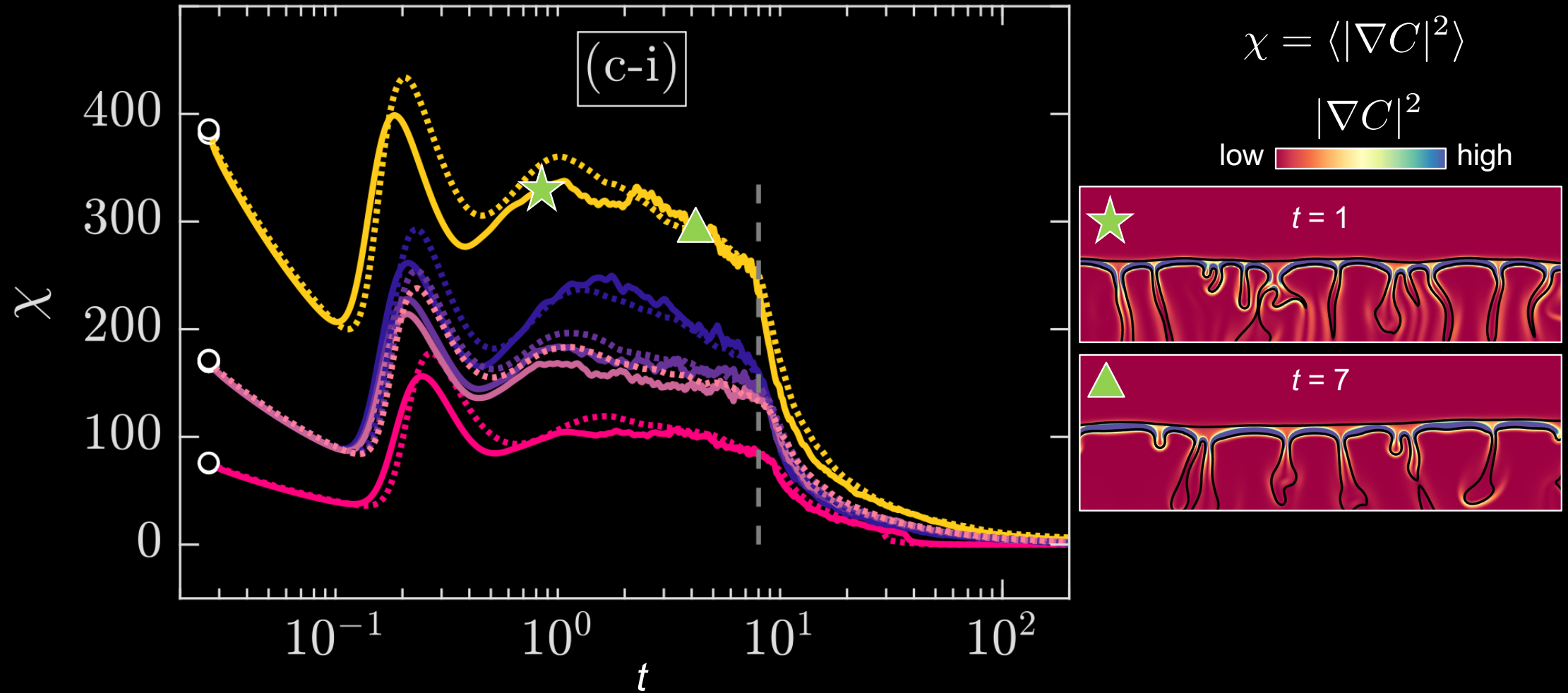


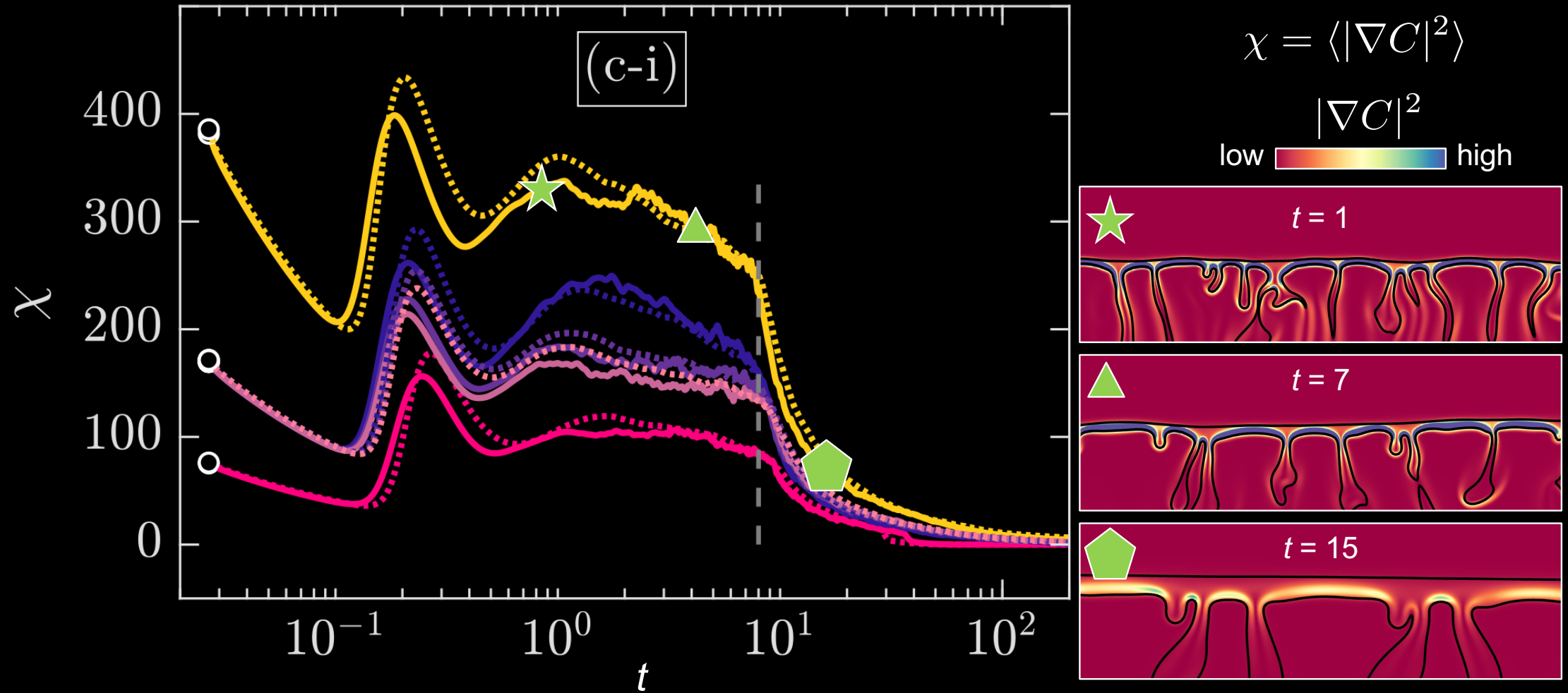
- Jha, B., Cueto-Felgueroso, L., & Juanes, R. (2011). *Phys. Rev. E*, 84 (6), 066312
 Hidalgo, J., Fe, J., Cueto-Felgueroso, L., & Juanes, R. (2012). *Phys. Rev. Lett.*, 109 (26), 264503.
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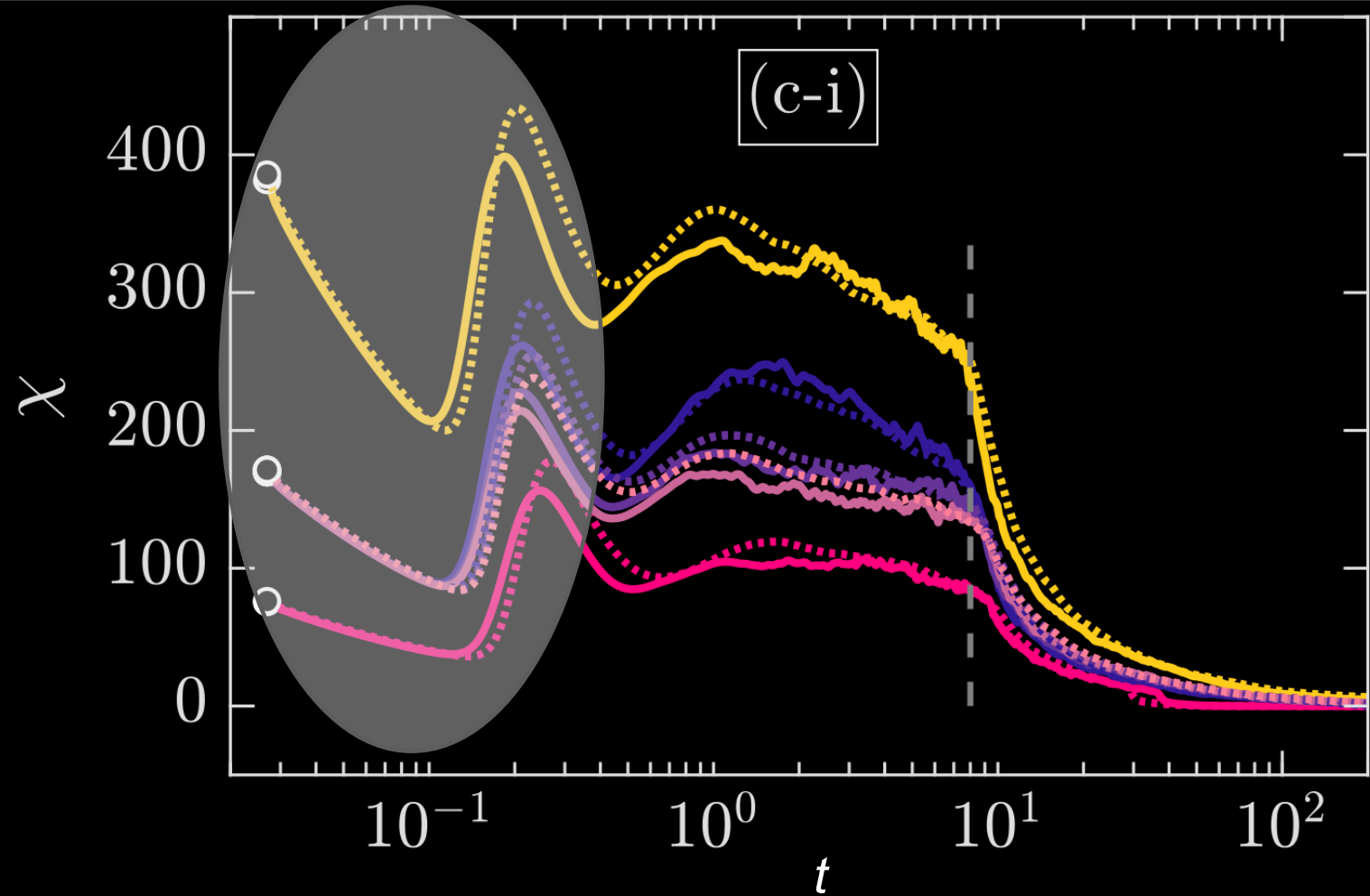


$$\chi = \langle |\nabla C|^2 \rangle$$

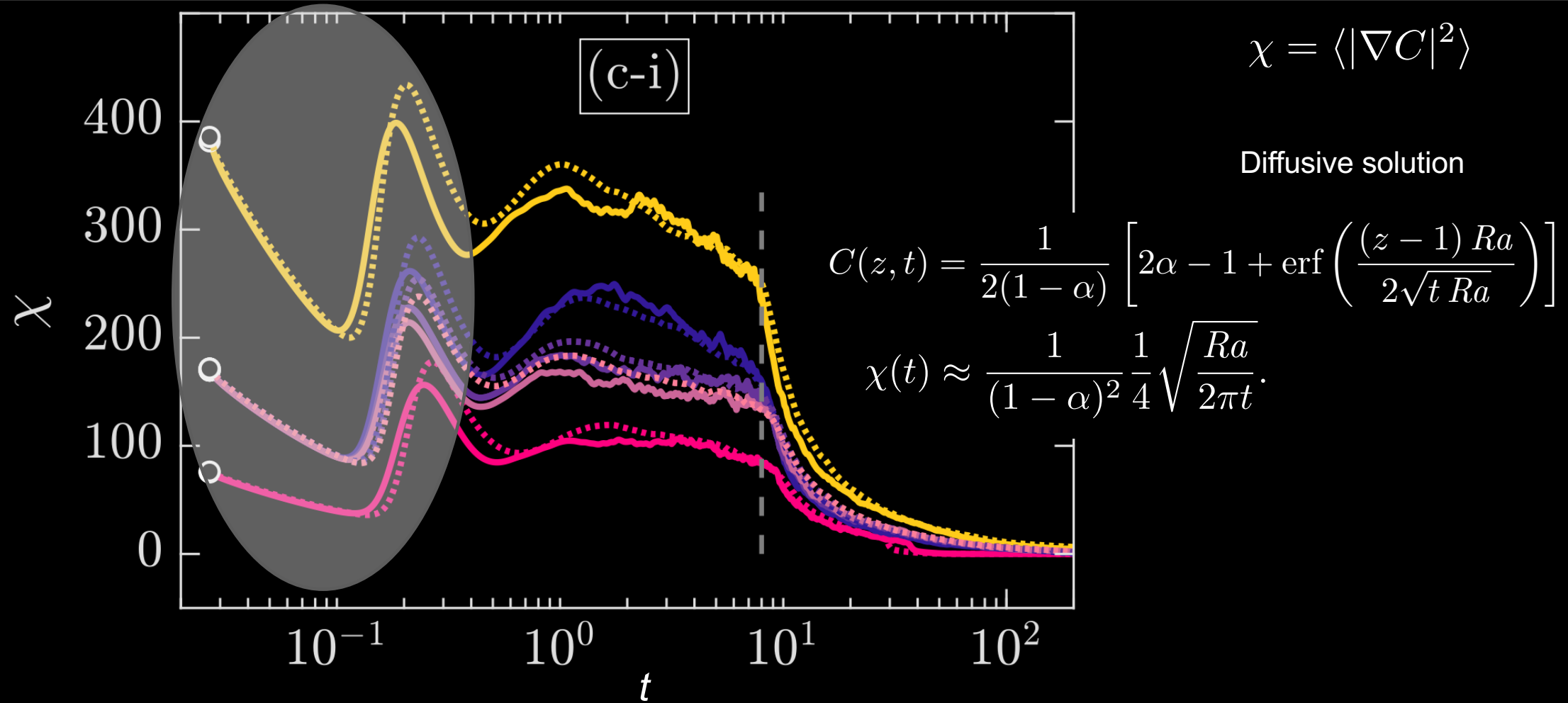


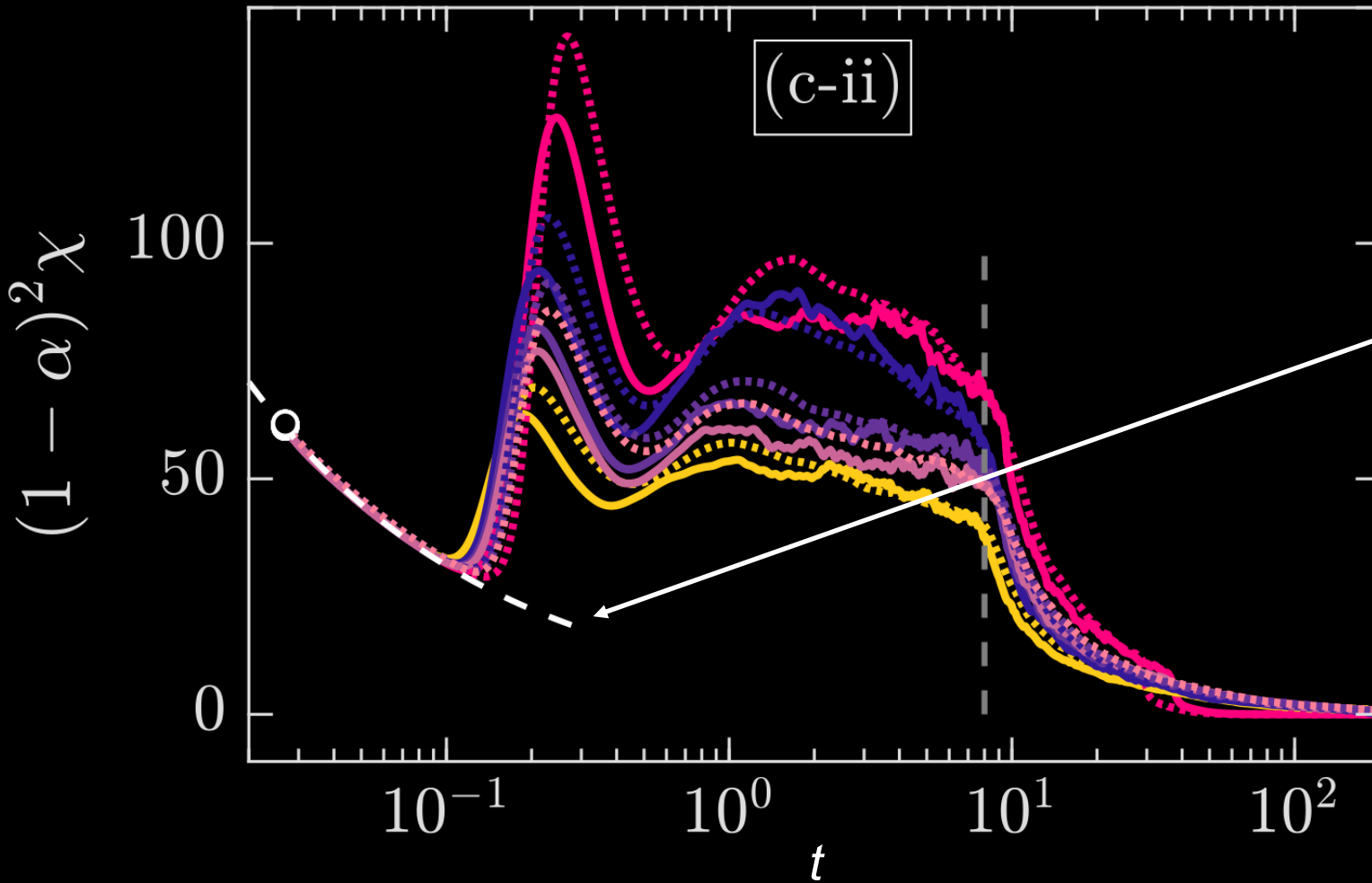




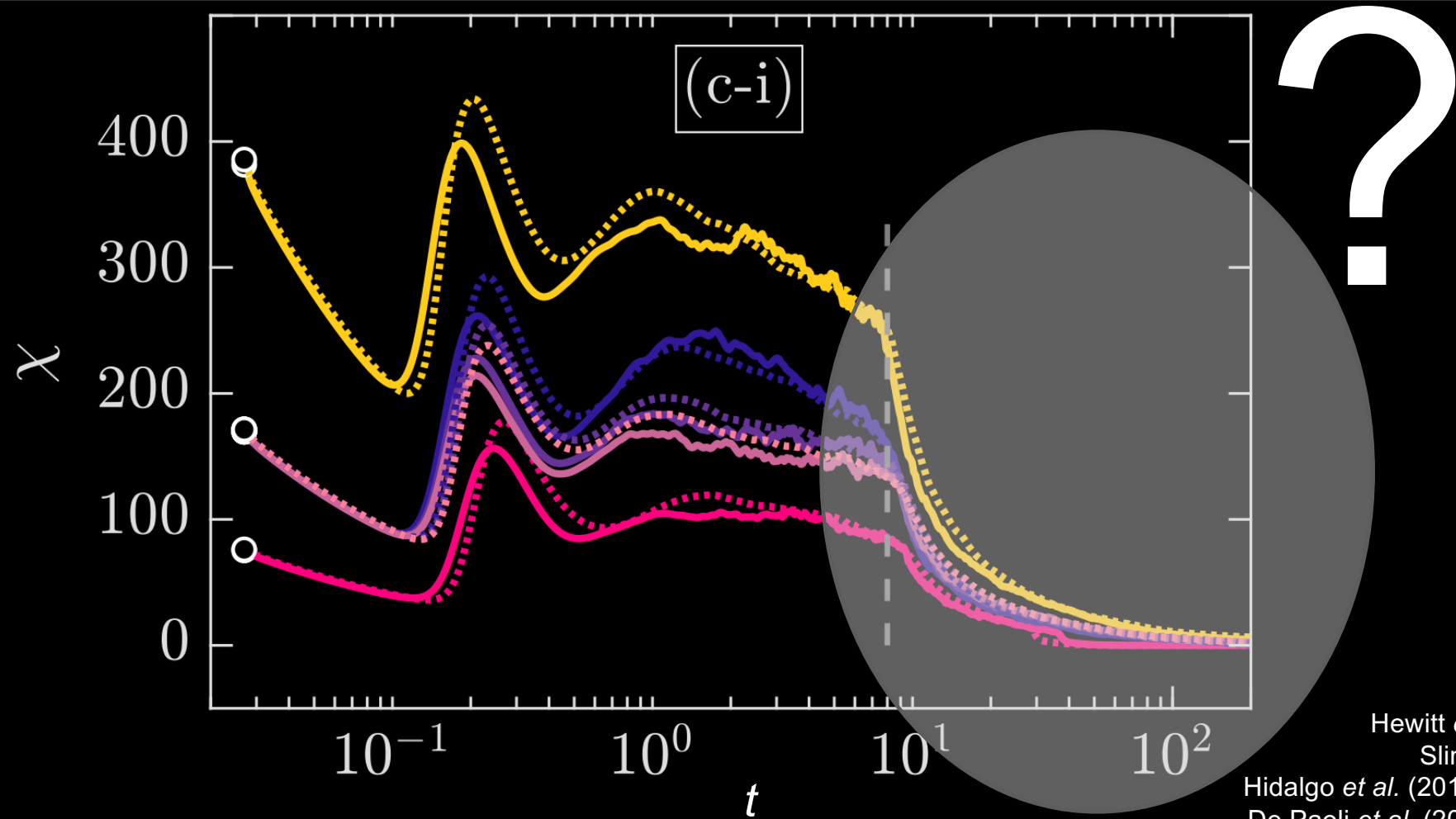


$$\chi = \langle |\nabla C|^2 \rangle$$



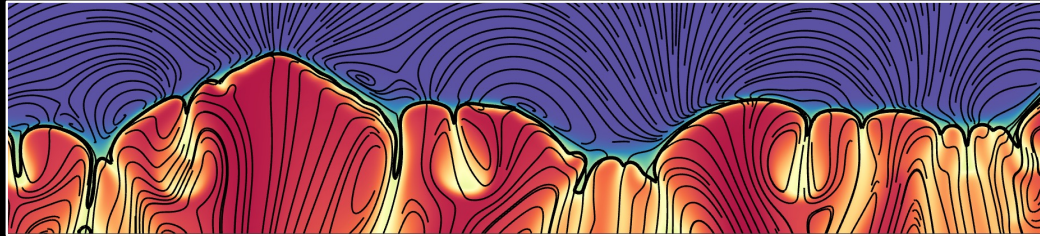


$$\chi(t) \approx \frac{1}{(1 - \alpha)^2} \frac{1}{4} \sqrt{\frac{Ra}{2\pi t}}$$

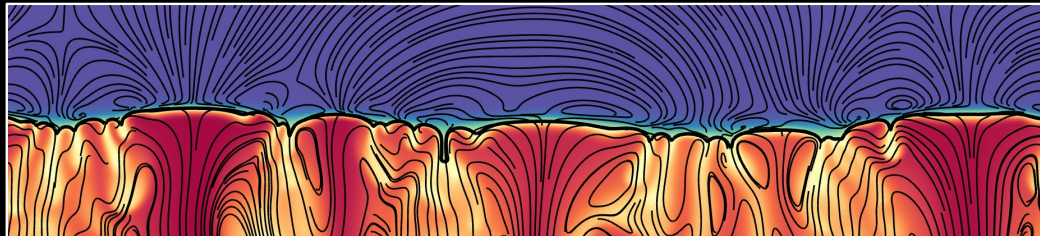


Hewitt et al. (2013). *J. Fluid Mech.*, 719
 Slim, A. (2014). *J. Fluid Mech.*, 741
 Hidalgo et al. (2015). *Geophys. Res. Lett.*, 42 (15)
 De Paoli et al. (2025). *Geophys. Res. Lett.*, 52 (7)

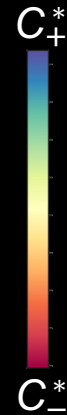
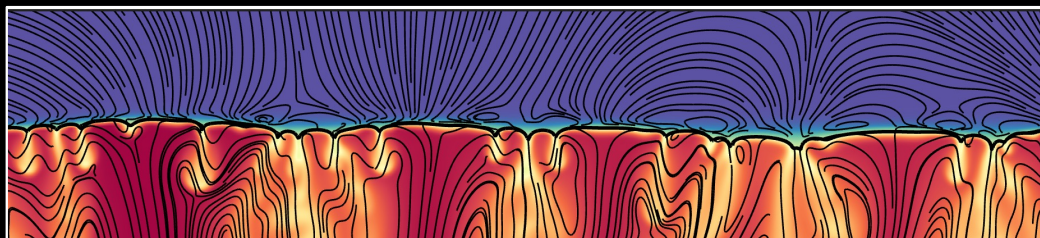
$\alpha = 0.4, \beta = 0.0$



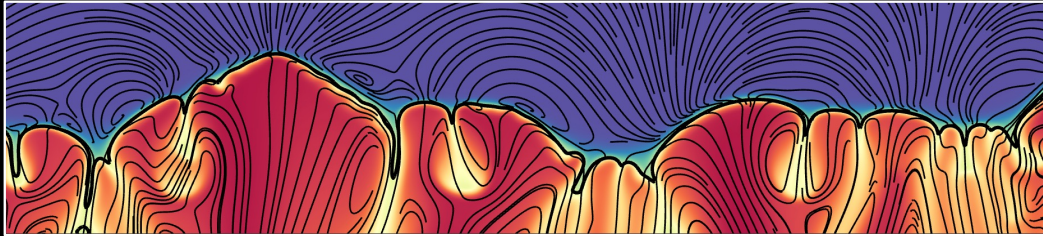
$\alpha = 0.4, \beta = 1.5$



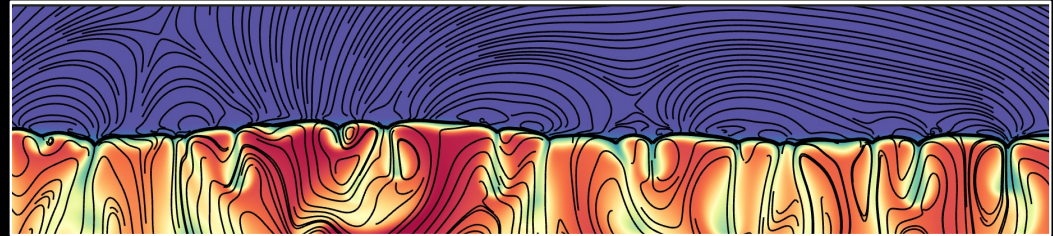
$\alpha = 0.4, \beta = 3.0$



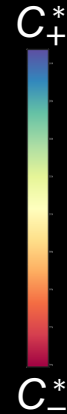
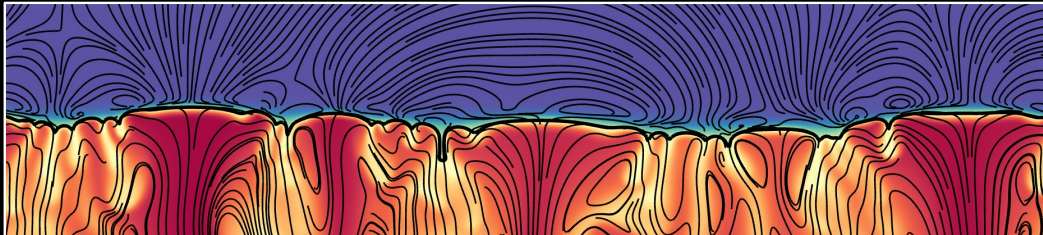
$\alpha = 0.4, \beta = 0.0$



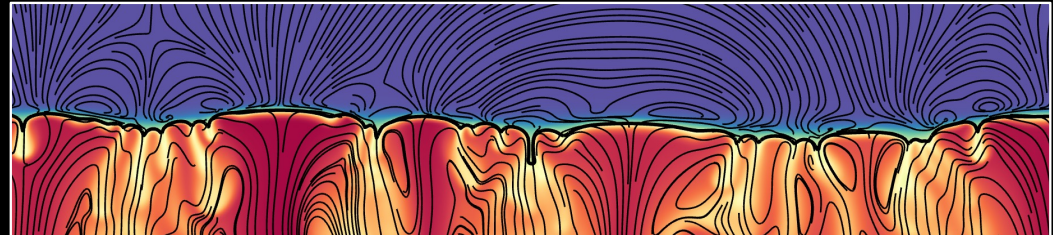
$\alpha = 0.1, \beta = 1.5$



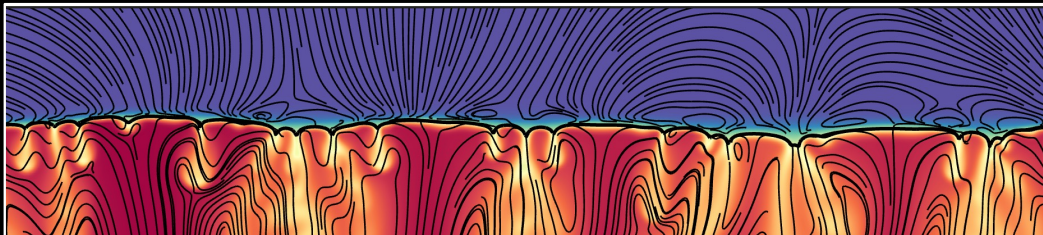
$\alpha = 0.4, \beta = 1.5$



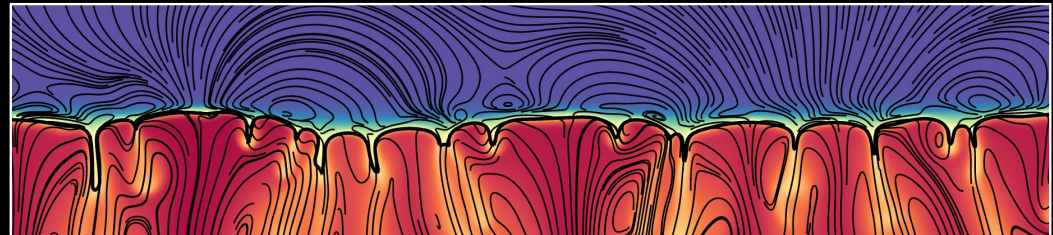
$\alpha = 0.4, \beta = 1.5$



$\alpha = 0.4, \beta = 3.0$



$\alpha = 0.6, \beta = 1.5$

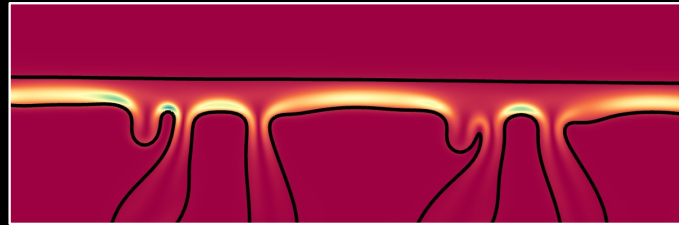


Model for dissipation during shutdown of convection

$$\chi = \langle |\nabla C|^2 \rangle$$

1. Estimate local dissipation $|\nabla C|^2$

$$|\partial_x C| \approx |\partial_y C| \ll |\partial_z C|$$



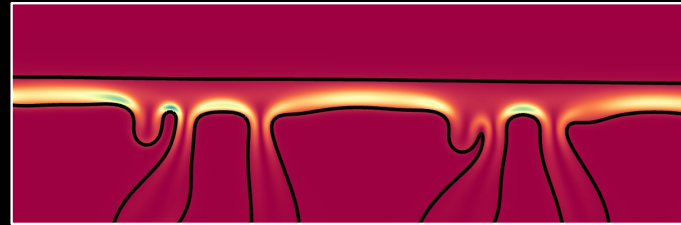
low  high

Model for dissipation during shutdown of convection

$$\chi = \langle |\nabla C|^2 \rangle$$

1. Estimate local dissipation $|\nabla C|^2$

$$|\partial_x C| \approx |\partial_y C| \ll |\partial_z C|$$



low  high

2. Estimate boundary layer thickness

$$\delta(t) = (C_+ - C_-) f(t) = \frac{f(t)}{1 - \alpha}$$

$$C_+ - C_- = 1/(1 - \alpha)$$

$$f(t) = [a_1 + (t - t_s)]/a_2$$

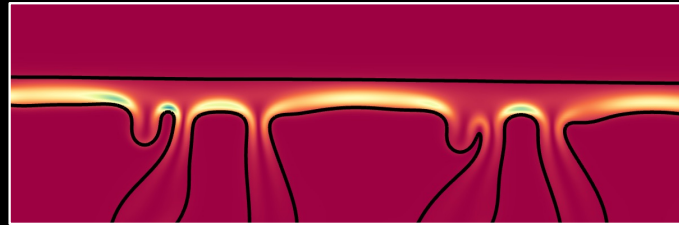
Interface deformation model
by Hidalgo et al. (2015)

Model for dissipation during shutdown of convection

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low  high

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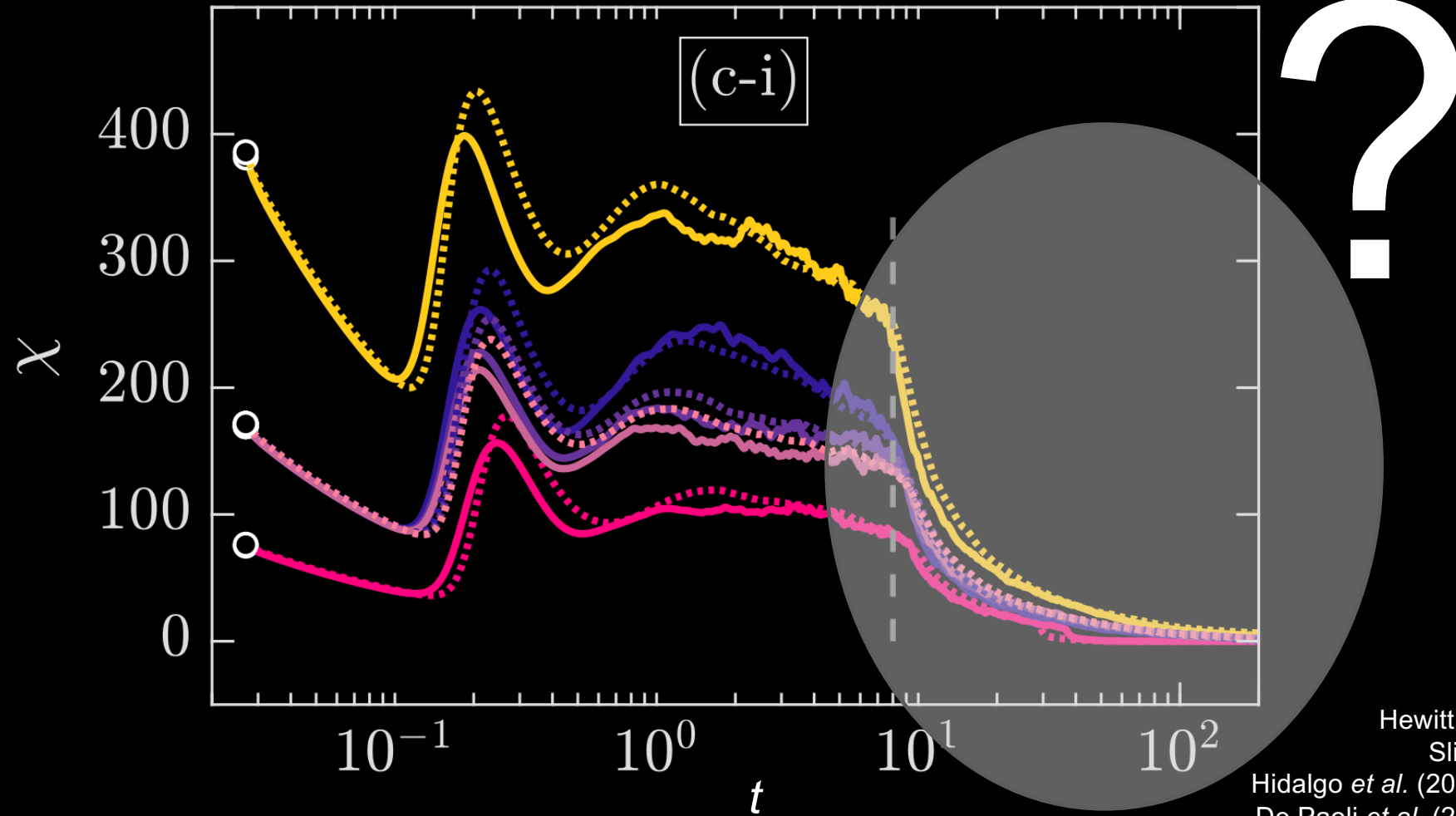
$$f(t) = [a_1 + (t - t_s)]/a_2$$

Interface deformation model
by Hidalgo et al. (2015)

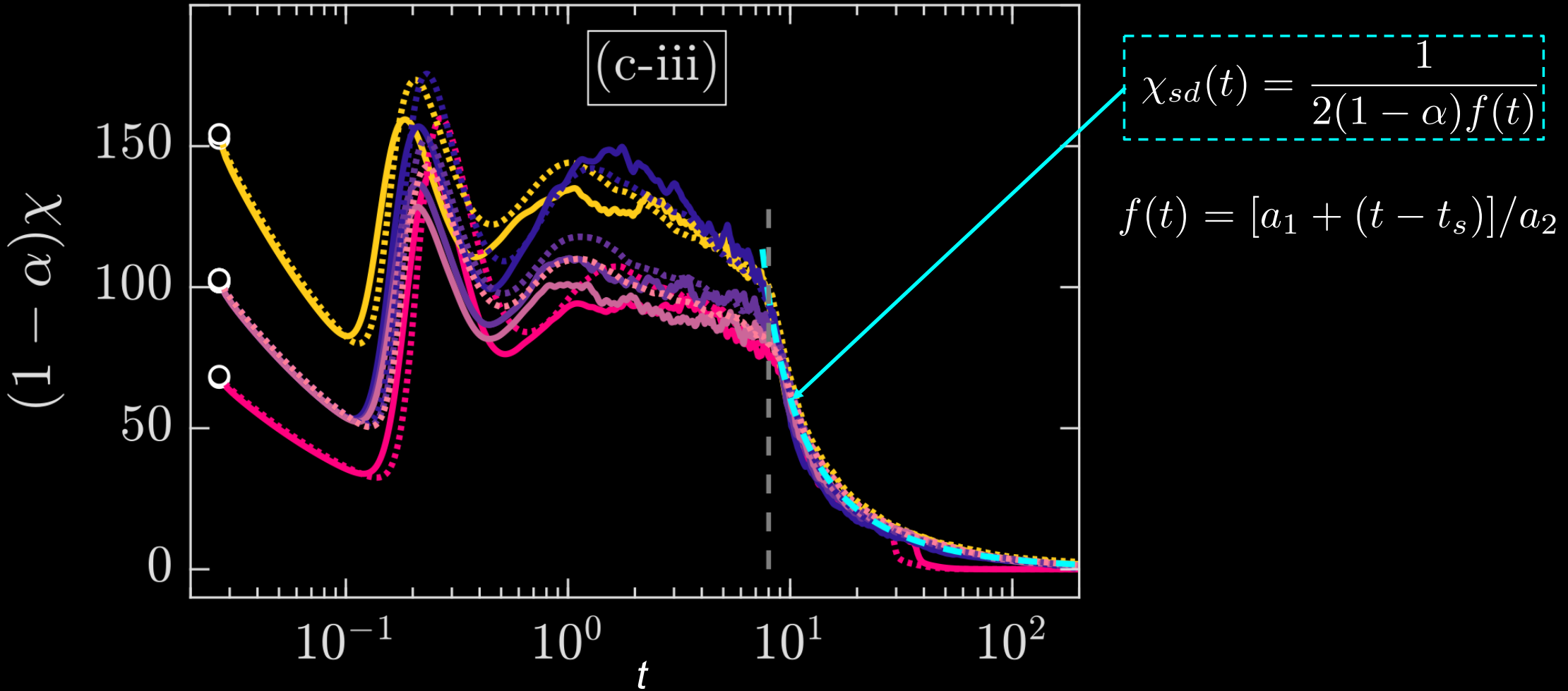
3. Combine and find dissipation

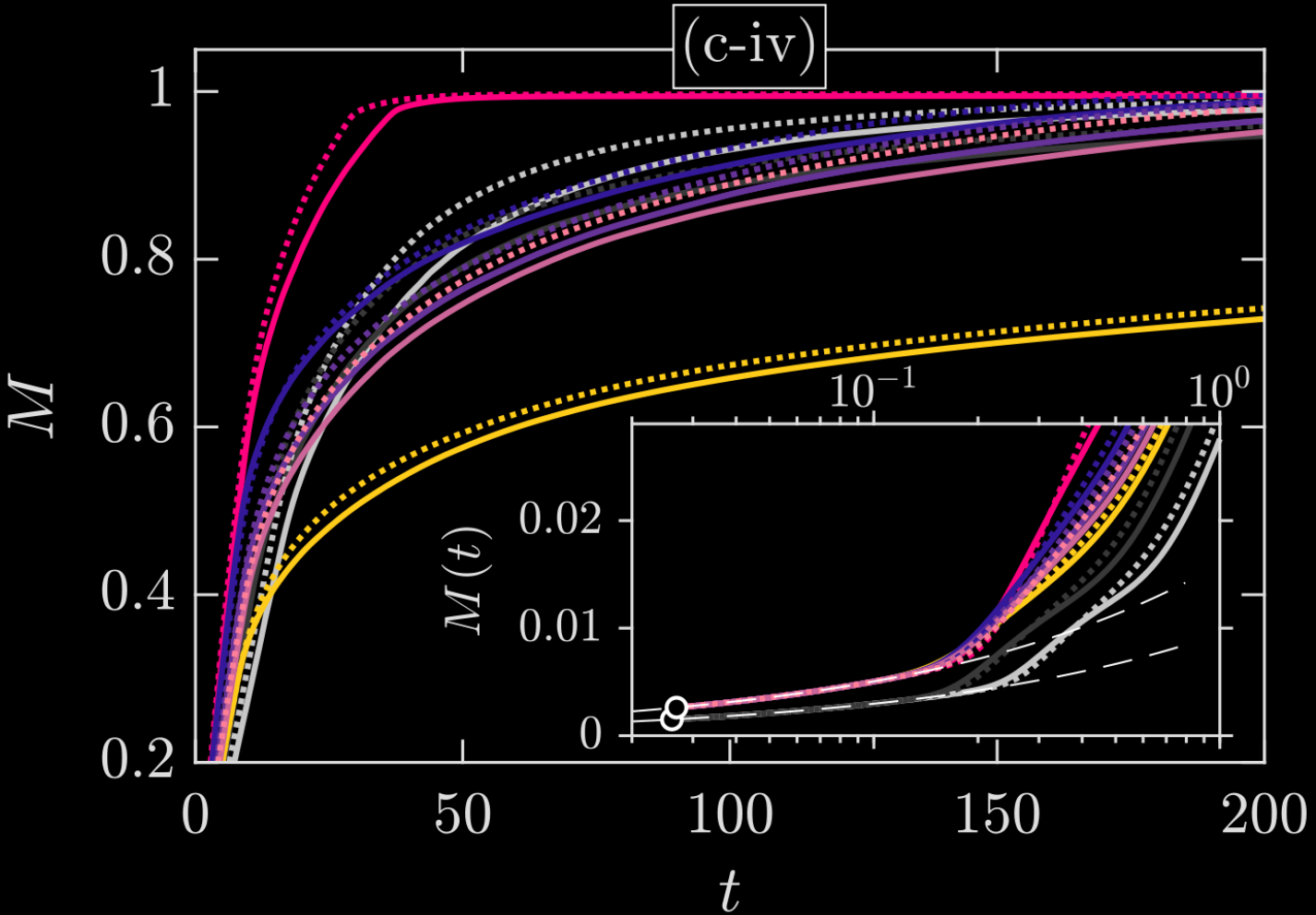
$$\chi = \langle |\nabla C|^2 \rangle \approx \frac{\delta}{L_z} \left(\frac{C_+ - C_-}{\delta} \right)^2$$

$$\chi_{sd}(t) = \frac{1}{2(1 - \alpha) f(t)}$$



Hewitt et al. (2013). *J. Fluid Mech.*, 719
 Slim, A. (2014). *J. Fluid Mech.*, 741
 Hidalgo et al. (2015). *Geophys. Res. Lett.*, 42 (15)
 De Paoli et al. (2025). *Geophys. Res. Lett.*, 52 (7)





$$M(t) = \frac{8(1 - \alpha)^2}{Ra} \int_0^t \chi dt$$

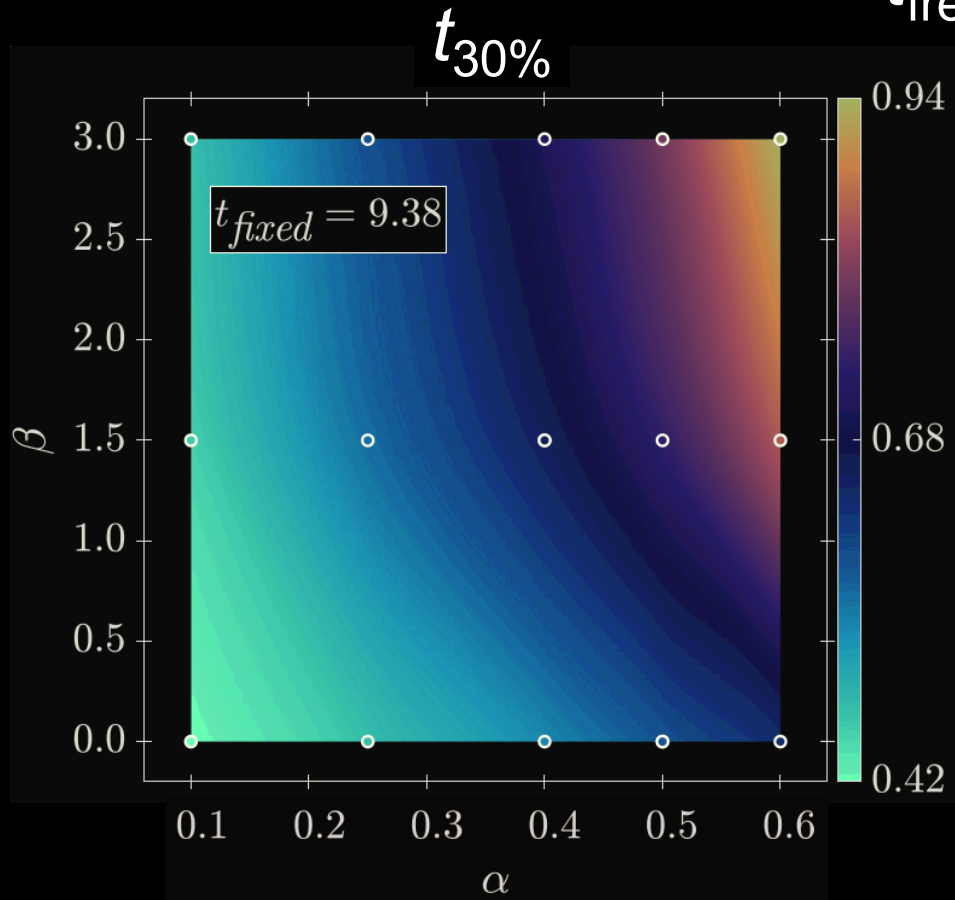
fixed interface

- A1 (2D) A3 (3D)
- A2 (2D) A4 (3D)

free interface

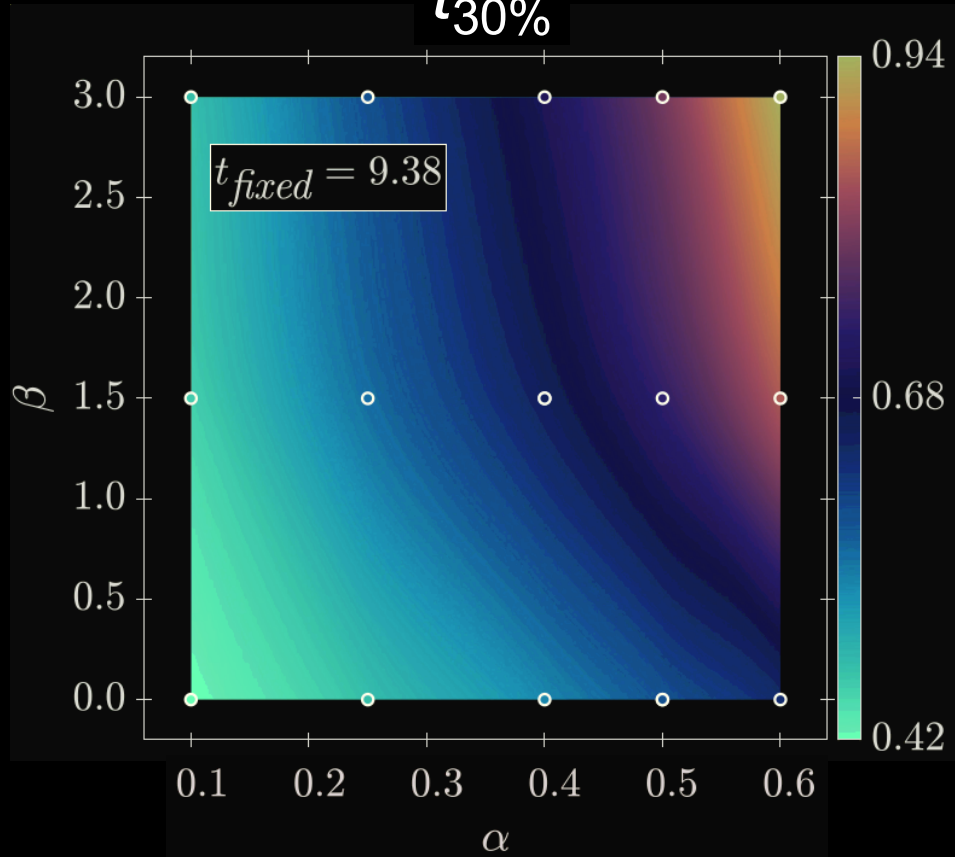
- B1 (2D) B4 (3D)
- B2 (2D) B5 (3D)
- B3 (2D) B6 (3D)
- C1 (2D) C4 (3D)
- C2 (2D) C5 (3D)
- C3 (2D) C6 (3D)

$t_{\text{free}} / t_{\text{fixed}}$ (3D, linear)

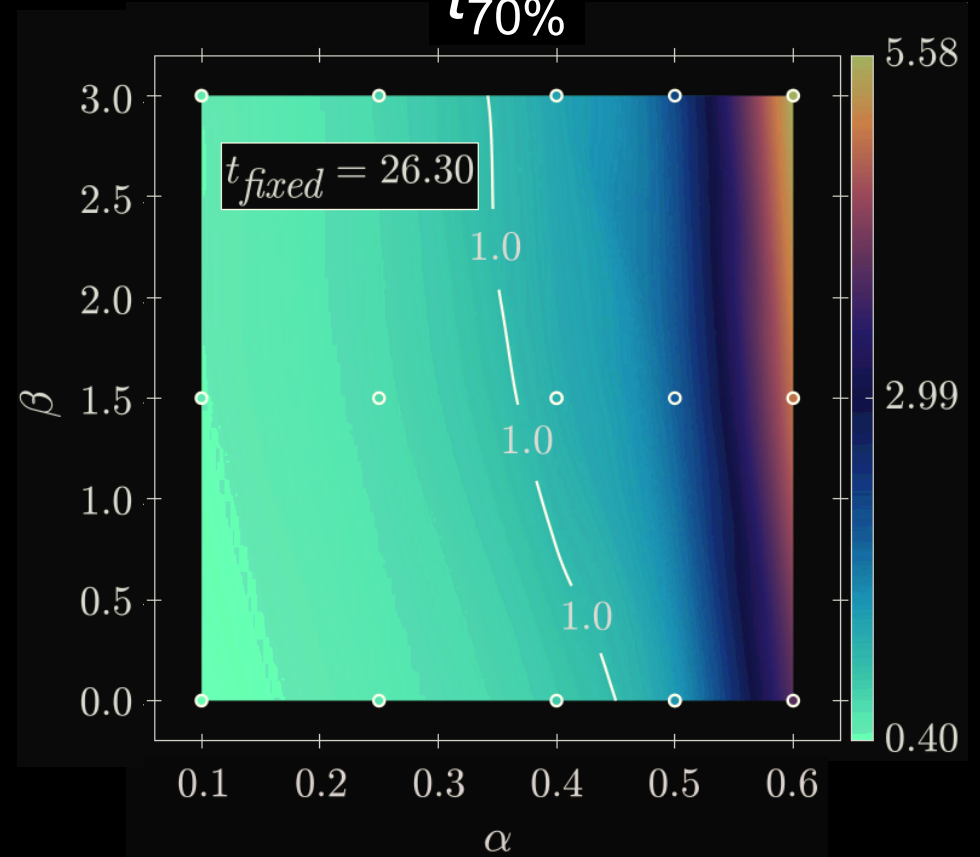


$t_{\text{free}} / t_{\text{fixed}}$ (3D, linear)

$t_{30\%}$



$t_{70\%}$



Theoretical framework for comparison of system with different b.c. and fluid models

De Paoli & Pirozzoli
(arXiv:2604.23199)

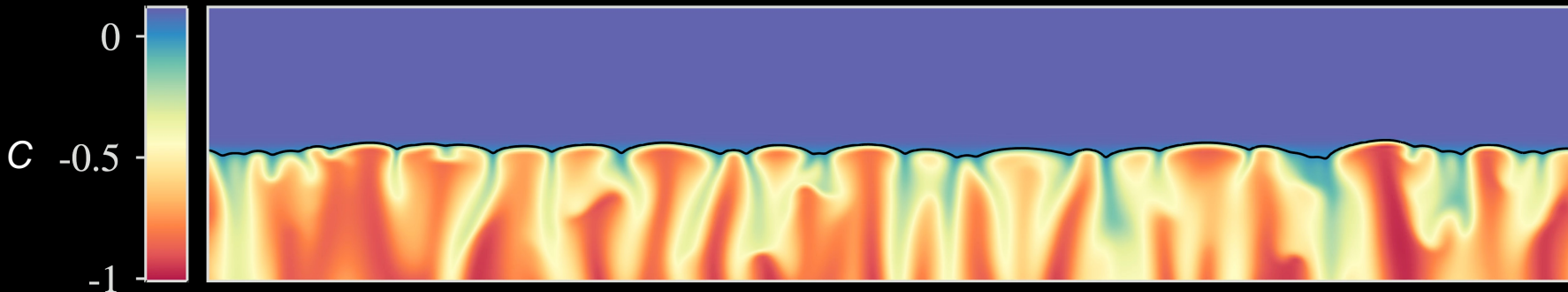


preprint



data

$\alpha = 0.1$
 $\beta = 1.5$



Theoretical framework for comparison of system with different b.c. and fluid models

Understanding and modelling of mixing dynamics

De Paoli & Pirozzoli
(arXiv:2604.23199)

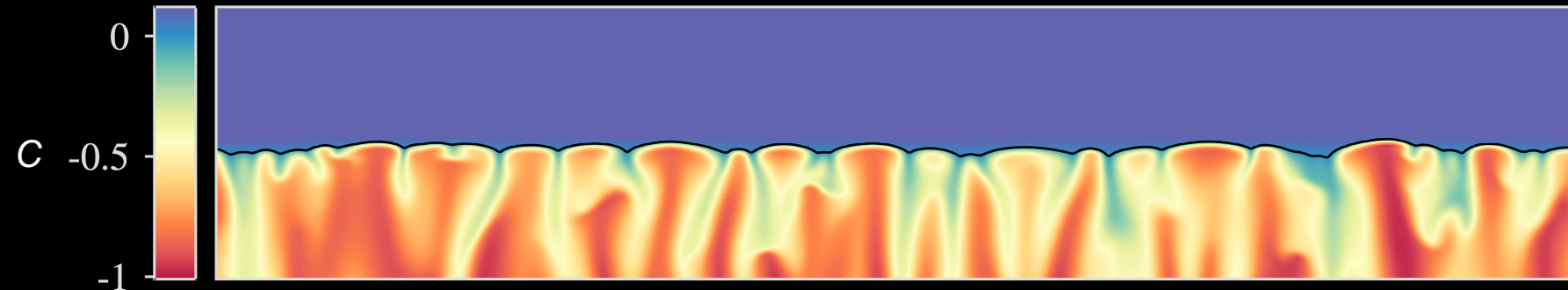


preprint



data

$\alpha = 0.1$
 $\beta = 1.5$



Theoretical framework for comparison of system with different b.c. and fluid models

Understanding and modelling of mixing dynamics

Comparison of different models and prediction of uncertainties/differences

De Paoli & Pirozzoli
(arXiv:2604.23199)

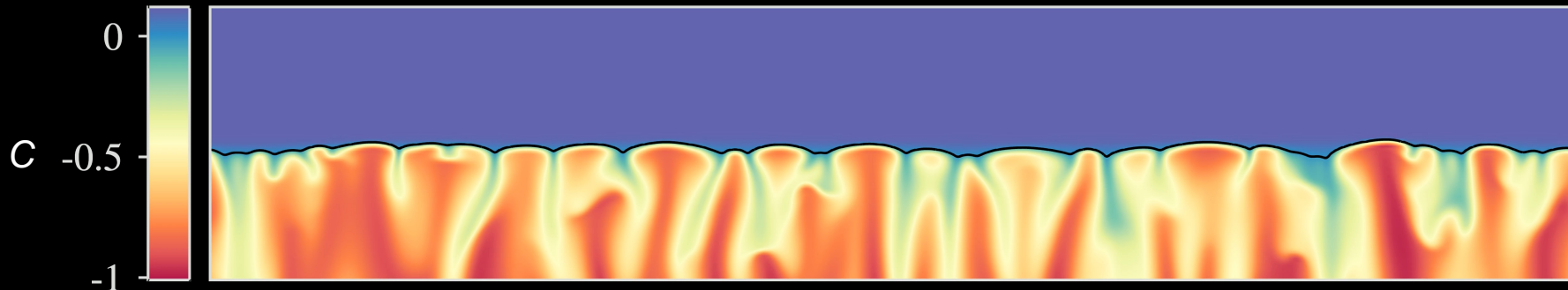


preprint



data

$\alpha = 0.1$
 $\beta = 1.5$



Theoretical framework for comparison of system with different b.c. and fluid models

Understanding and modelling of mixing dynamics

Comparison of different models and prediction of uncertainties/differences



Funded by the European Union



European Research Council
Established by the European Commission



De Paoli & Pirozzoli
(arXiv:2604.23199)

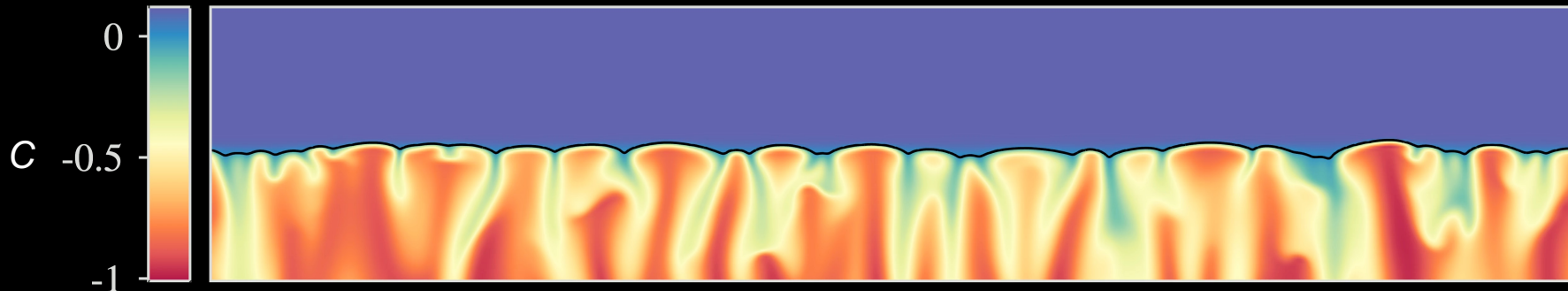


preprint



data

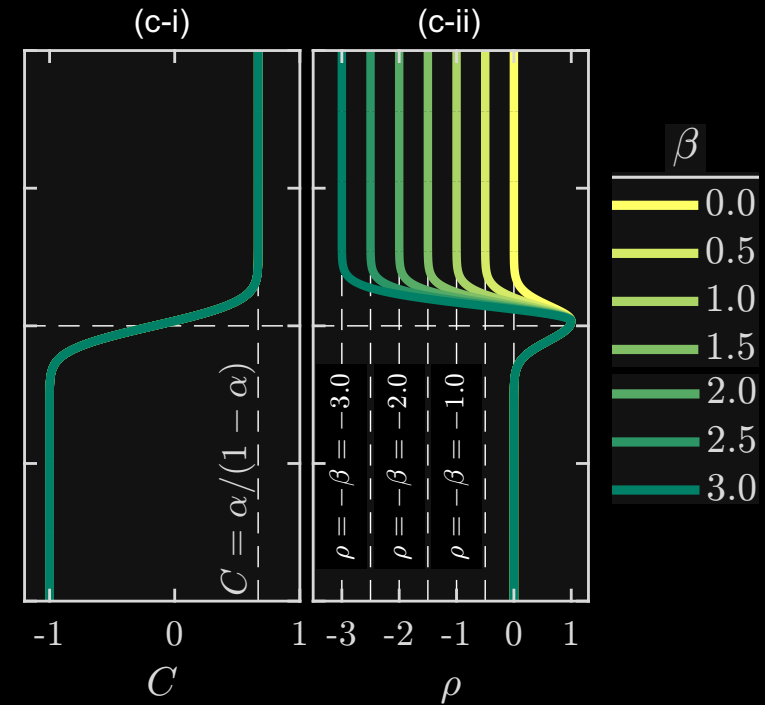
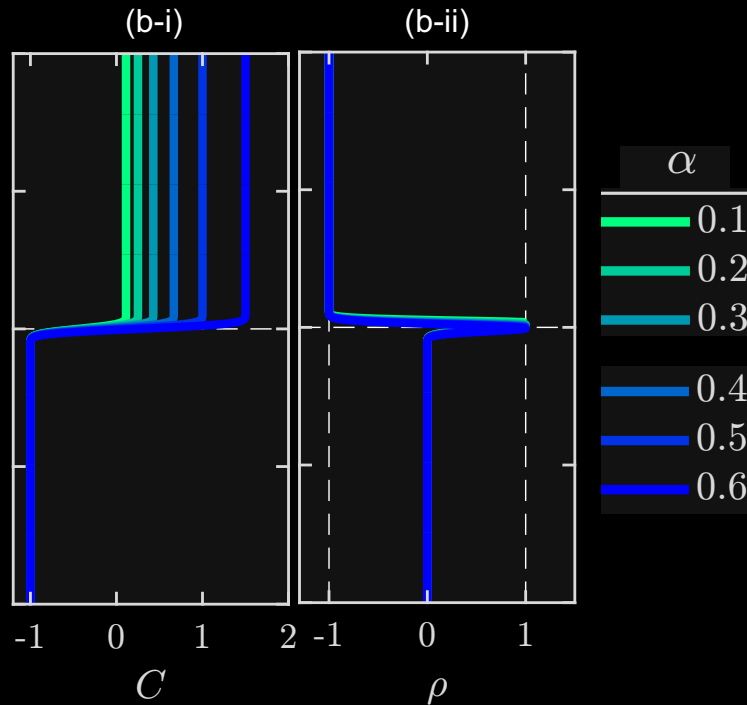
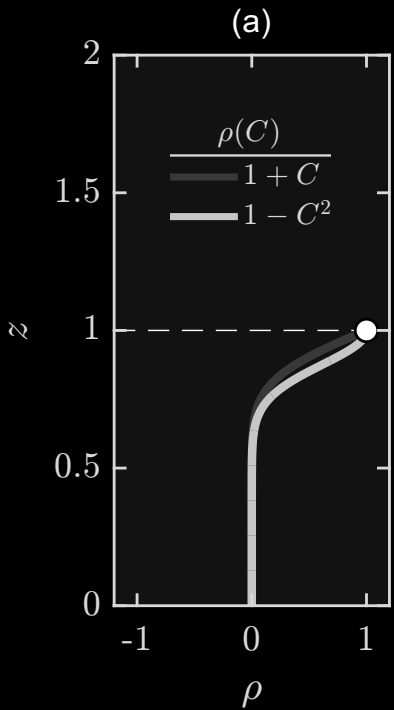
Funded by the European Union (ERC, MORPHOS, 101163625). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Research Council. Neither the European Union nor the granting authority can be held responsible for them. The results presented have been achieved using in part the Vienna Scientific Cluster (VSC).



$\alpha = 0.1$
 $\beta = 1.5$

fixed interface

free interface

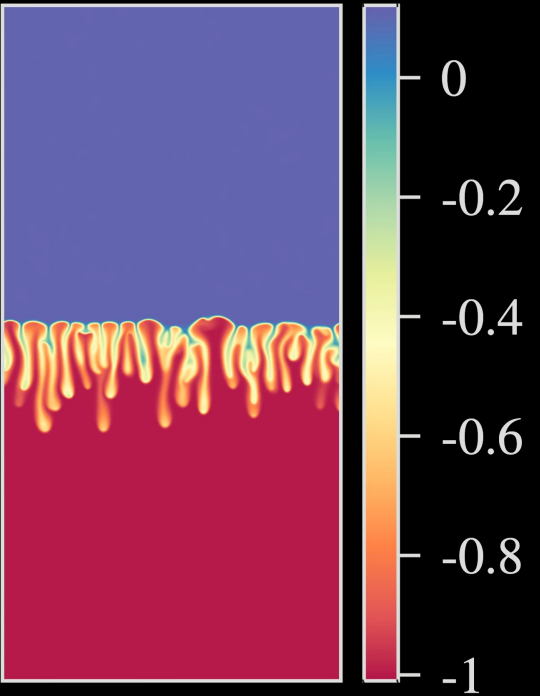


$$\alpha = \frac{C_+^* - C_m^*}{C_+^* - C_-^*}, \quad \beta = \frac{\rho_-^* - \rho_+^*}{\rho_m^* - \rho_-^*}$$

$\alpha = 0.1, \beta = 0.0$

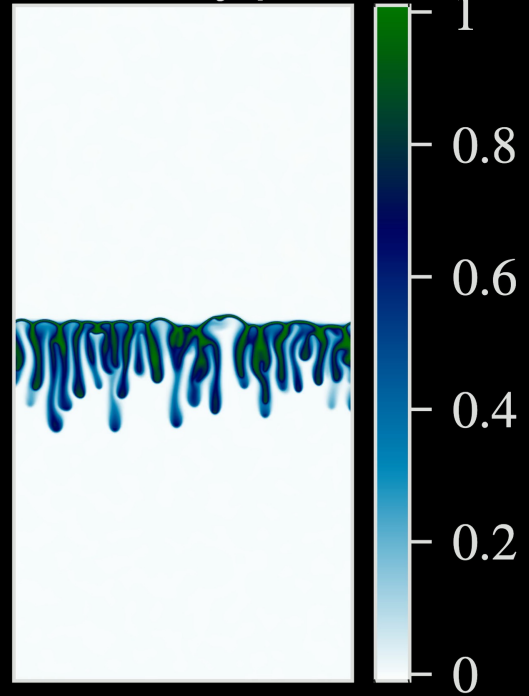
$t = 1.0$

concentration C

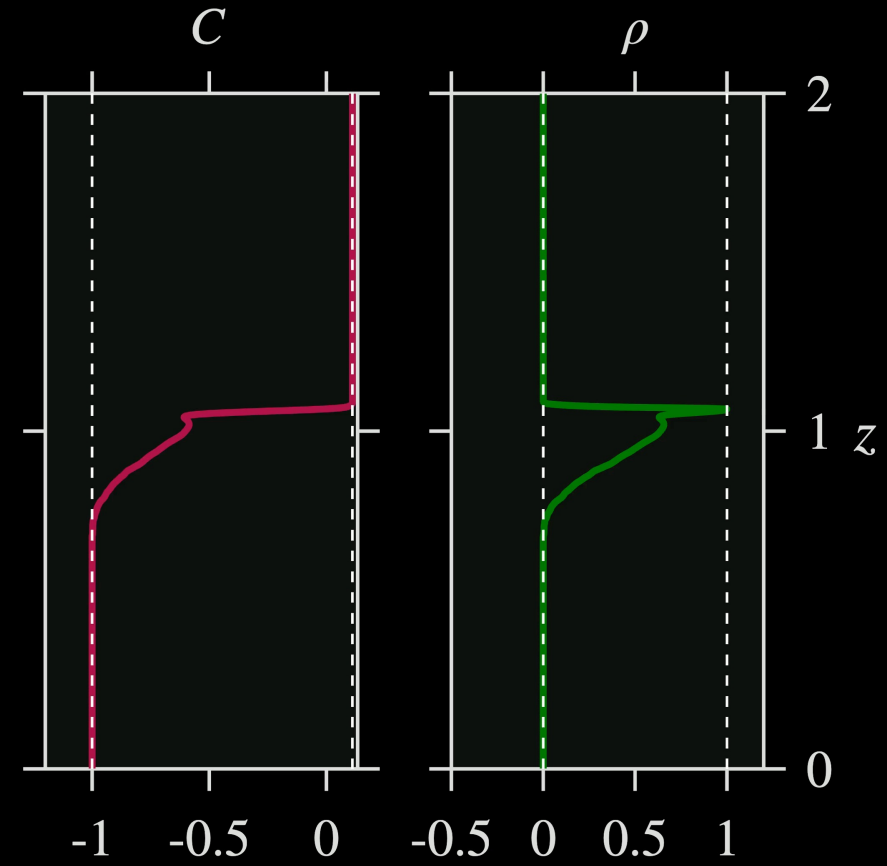


$C_{max} = 0.1$

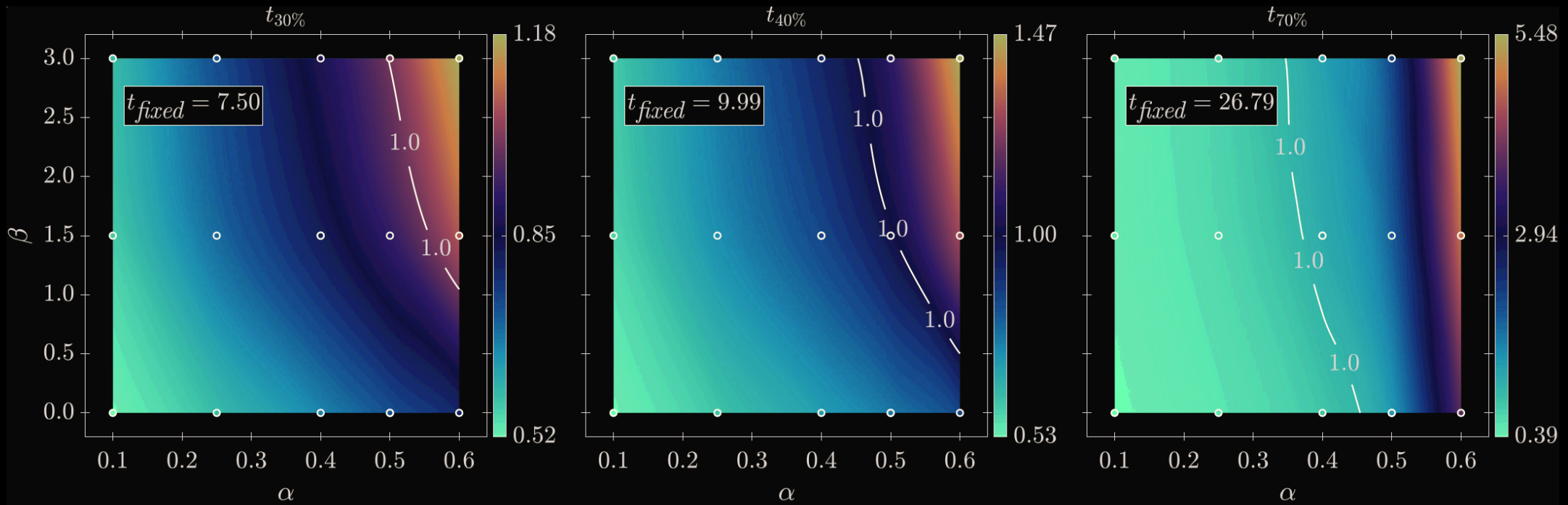
density ρ

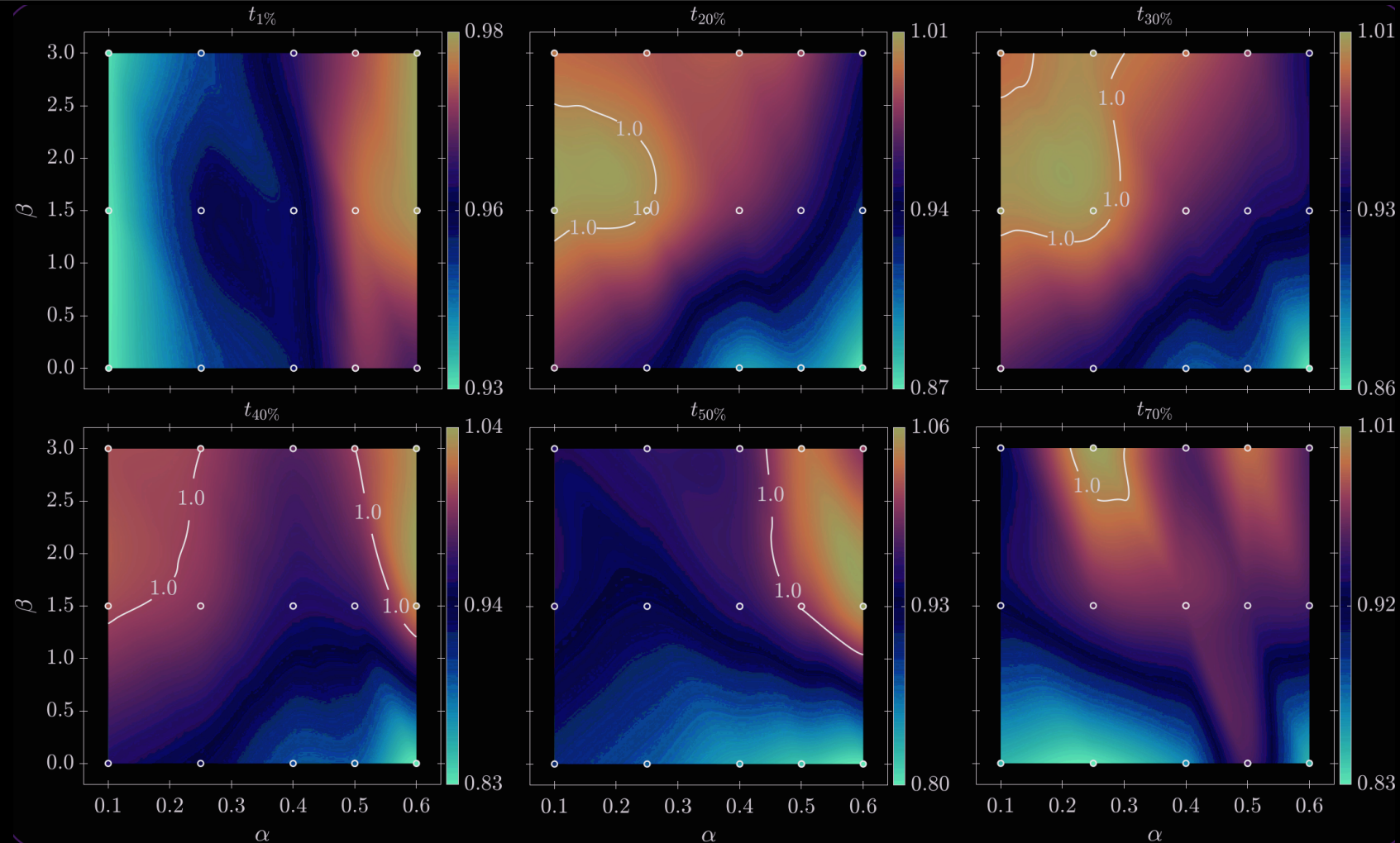


$\rho_{min} = -0.0$

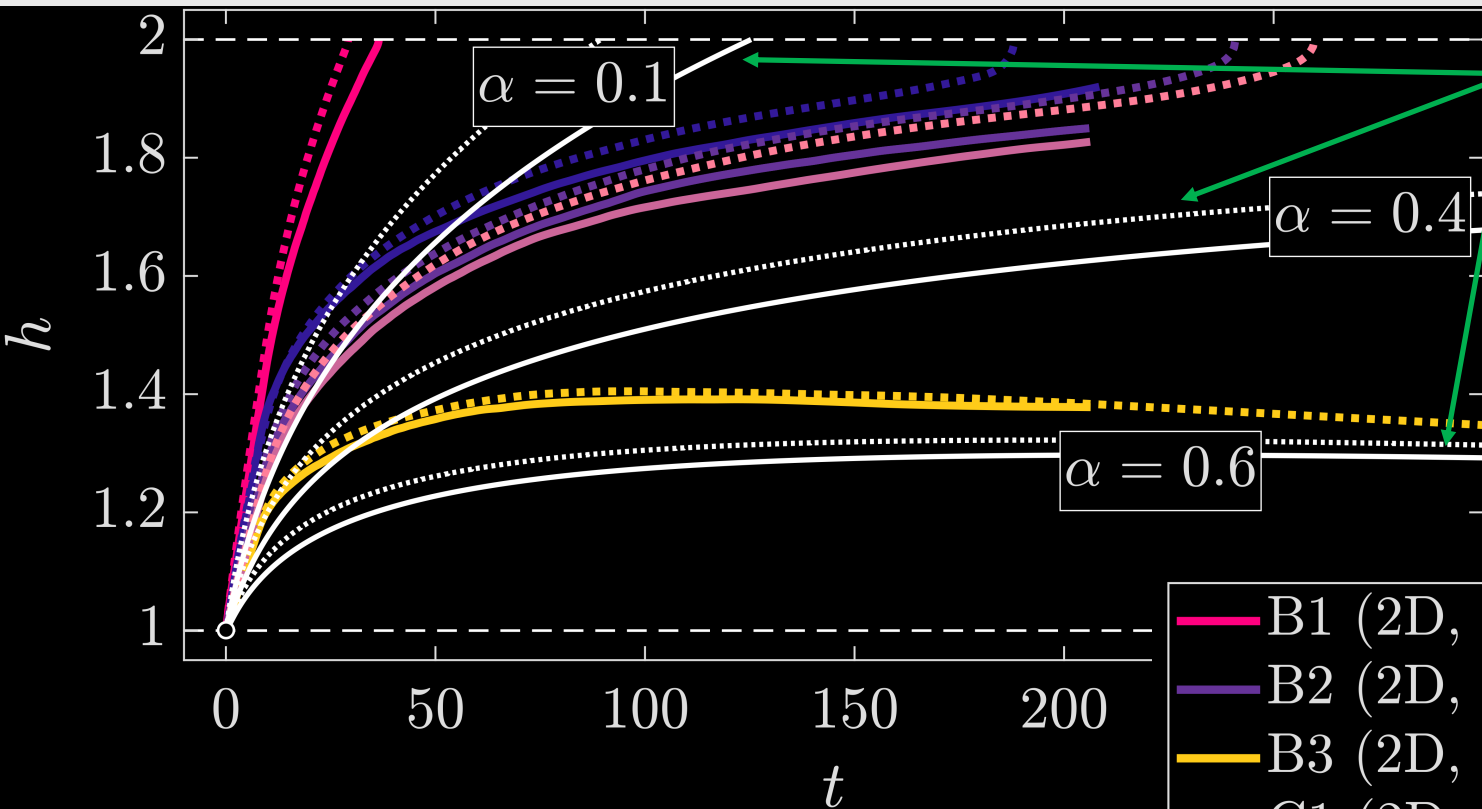


$t_{\text{free}} / t_{\text{fixed}}$ (3D, parabolic)





t_{2D} / t_{3D}
(free interface)



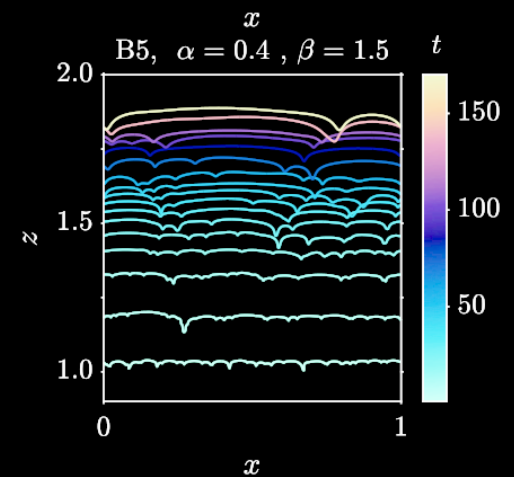
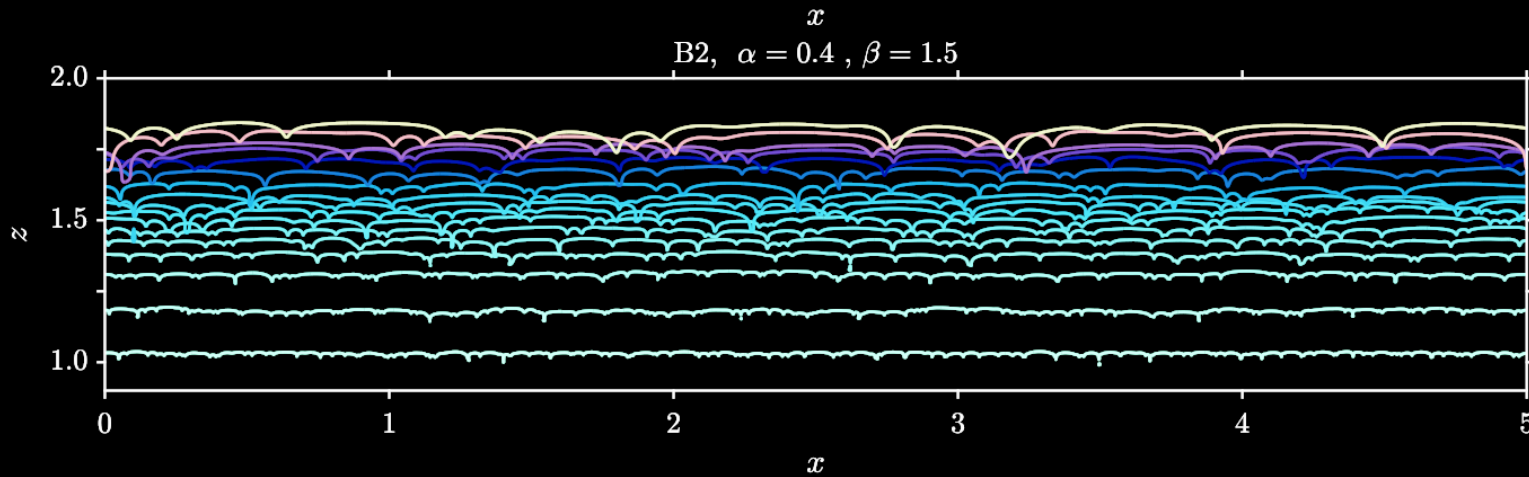
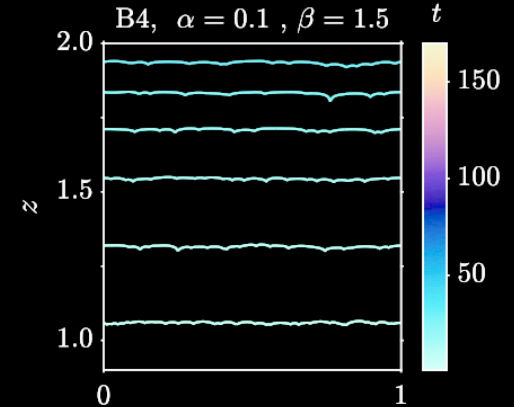
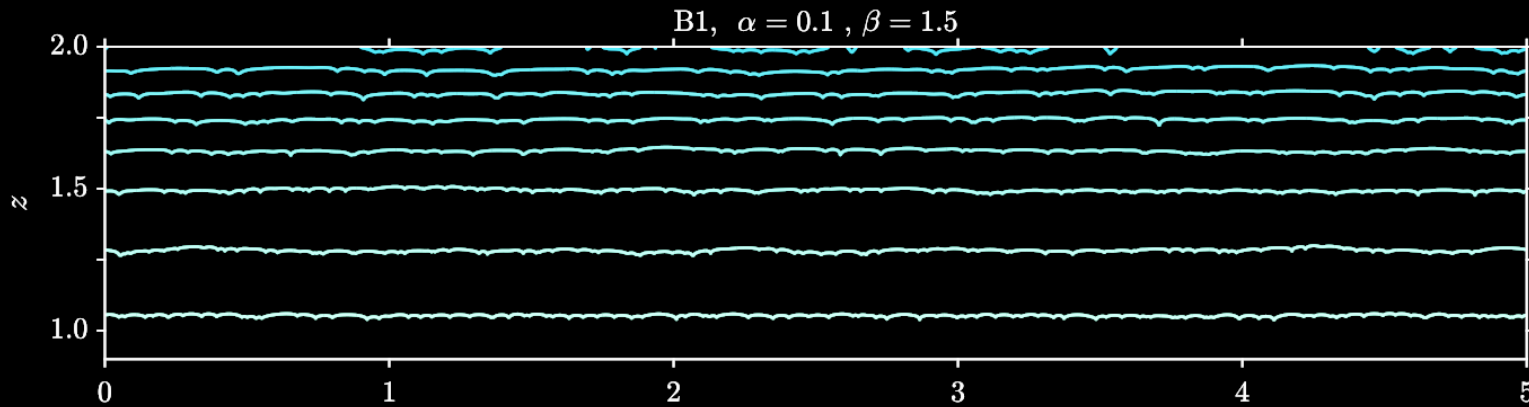
Model of Hewitt et al. (2013)

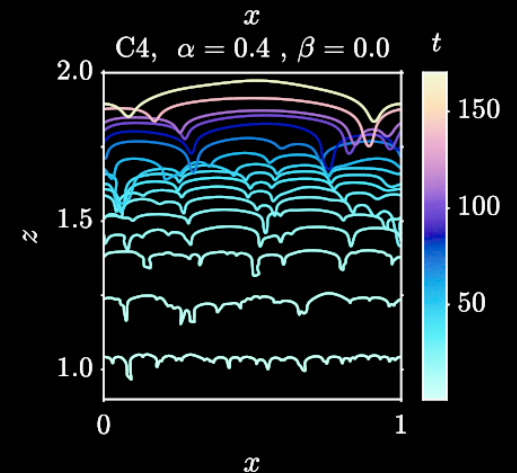
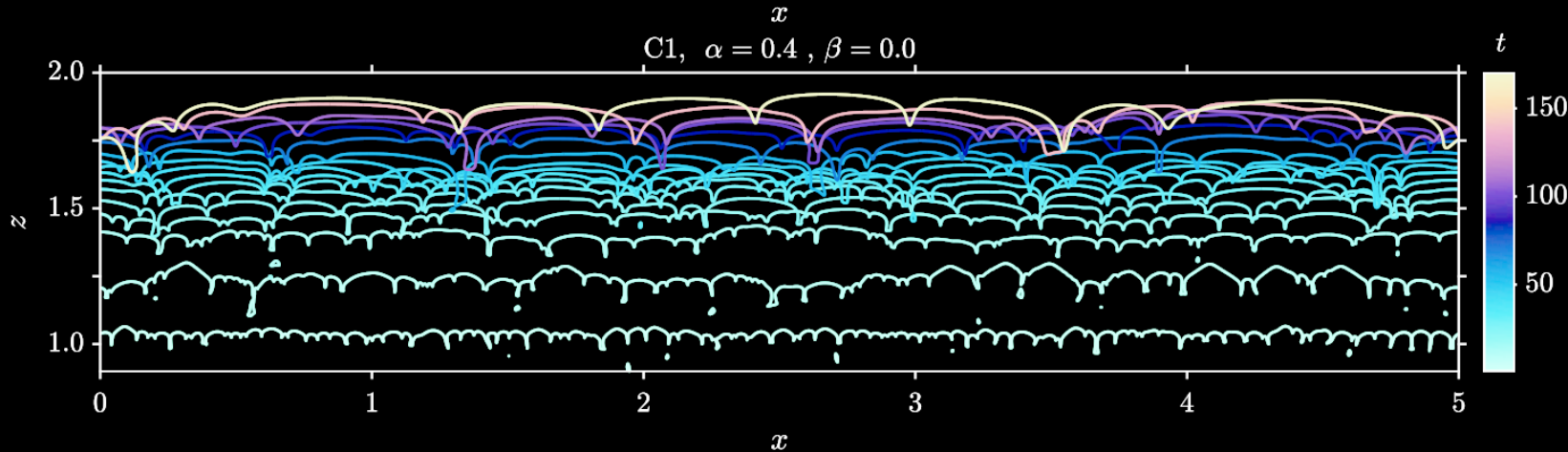
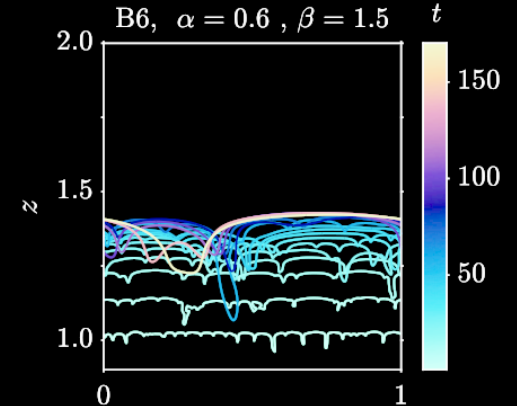
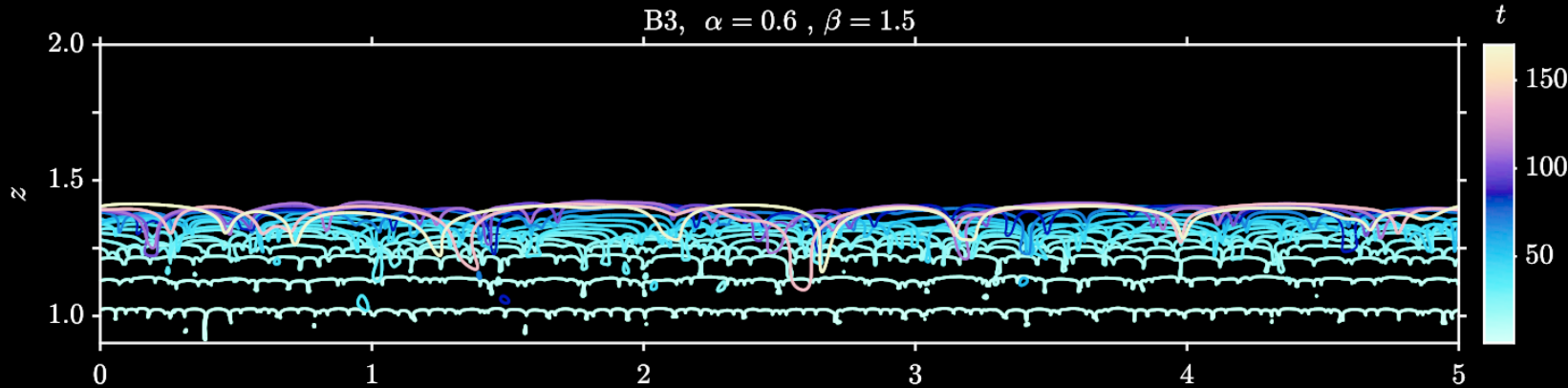
$$h \frac{d\Theta}{dt} = \frac{|\Theta|}{h Ra} Nu_e$$

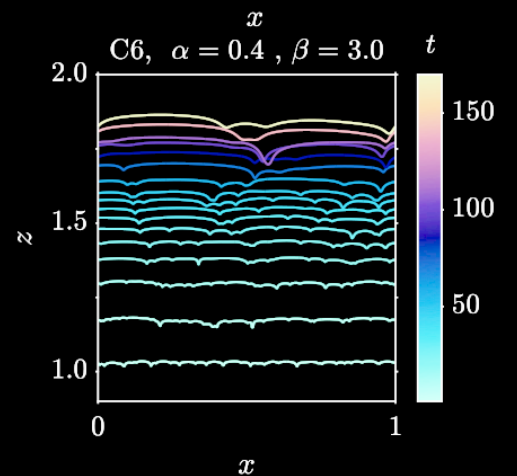
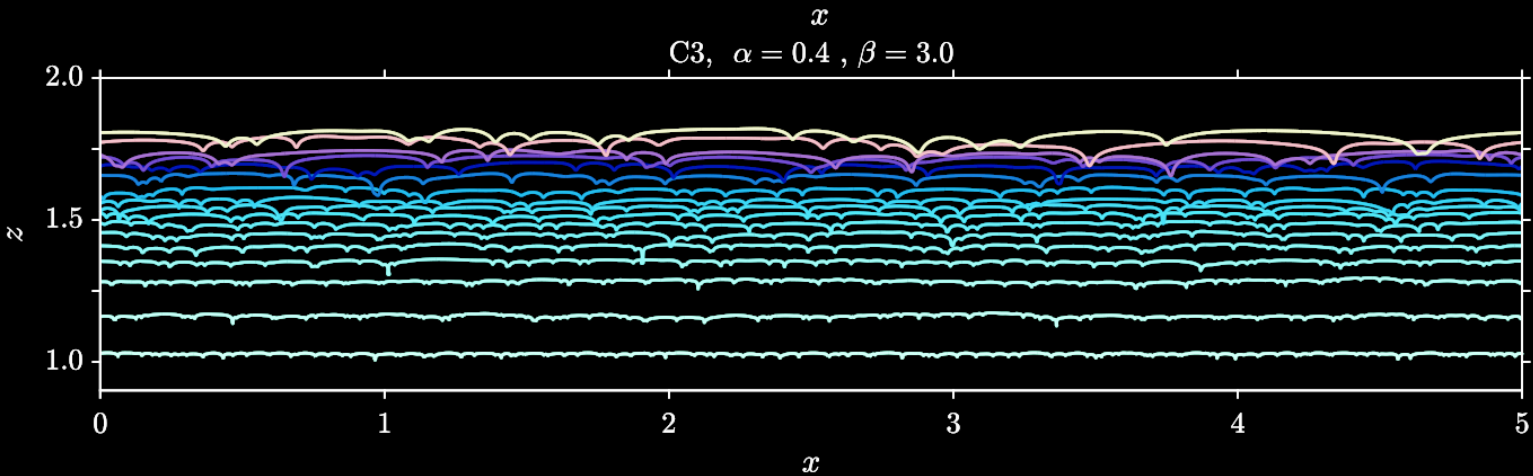
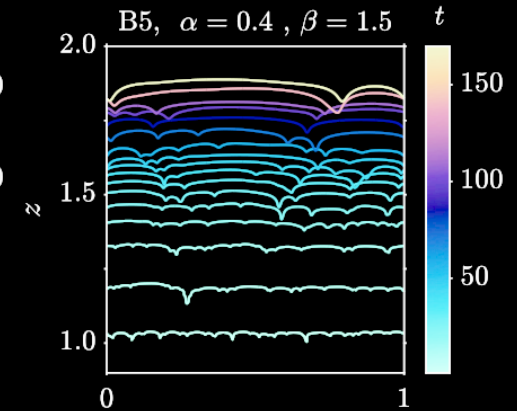
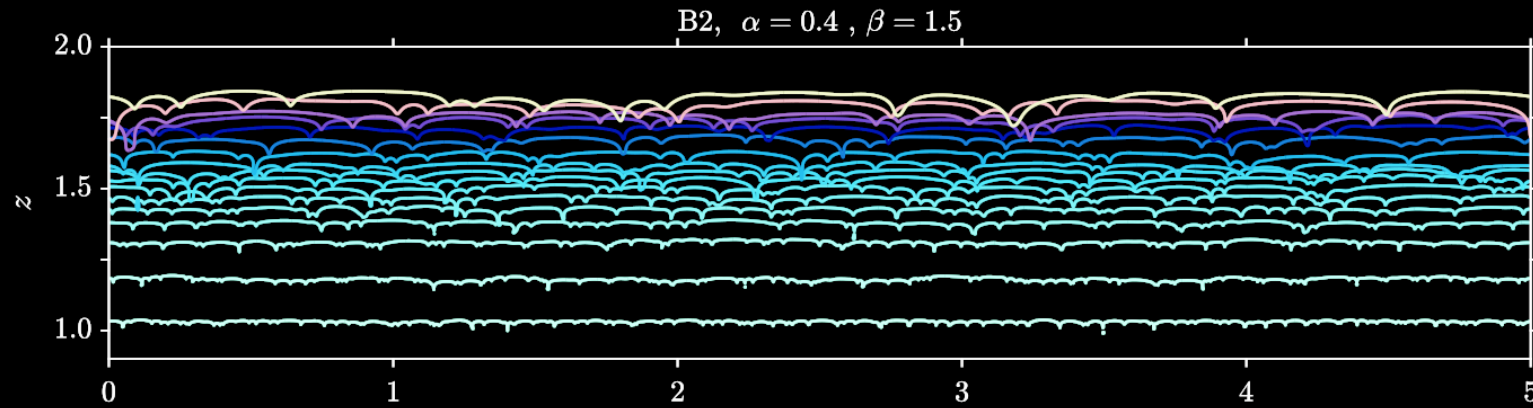
$$\left(|\Theta| + C_+ - \frac{2C_+}{\pi} \right) \frac{dh}{dt} = -\frac{4C_+}{\pi \delta Ra} + \frac{|\Theta| Nu_e}{h Ra}$$

$$\left(|\Theta| + C_+ - \frac{2C_+}{\pi} \right) \frac{d\delta}{dt} = \frac{8(C_+ + \Theta)}{\pi \delta Ra} - \frac{4|\Theta| Nu_e}{\pi h Ra}$$

- | | |
|----------------------------|----------------------------|
| — B1 (2D, $\alpha = 0.1$) | ⋯ B4 (3D, $\alpha = 0.1$) |
| — B2 (2D, $\alpha = 0.4$) | ⋯ B5 (3D, $\alpha = 0.4$) |
| — B3 (2D, $\alpha = 0.6$) | ⋯ B6 (3D, $\alpha = 0.6$) |
| — C1 (2D, $\alpha = 0.4$) | ⋯ C4 (3D, $\alpha = 0.4$) |
| — C2 (2D, $\alpha = 0.4$) | ⋯ C5 (3D, $\alpha = 0.4$) |
| — C3 (2D, $\alpha = 0.4$) | ⋯ C6 (3D, $\alpha = 0.4$) |







Sim.	interface/b.c.	Ra	$\rho(C)$	α	β	L_x	L_y	L_z	N_x	N_y	N_z
A1	fixed/(8)	1×10^4	linear (S6)	-	-	5	-	1	5120	1	256
A2	fixed/(8)	1×10^4	parabolic (S7)	-	-	5	-	1	5120	1	256
A3	fixed/(8)	1×10^4	linear (S6)	-	-	1	1	1	1024	1024	256
A4	fixed/(8)	1×10^4	parabolic (S7)	-	-	1	1	1	1024	1024	256
B1	free/(9)	1×10^4	piecewise (S8)	0.1	1.5	5	-	2	5120	1	1024
B2≡C2	free/(9)	1×10^4	piecewise (S8)	0.4	1.5	5	-	2	5120	1	1024
B3	free/(9)	1×10^4	piecewise (S8)	0.6	1.5	5	-	2	5120	1	1024
B4	free/(9)	1×10^4	piecewise (S8)	0.1	1.5	1	1	2	1024	1024	1024
B5≡C5	free/(9)	1×10^4	piecewise (S8)	0.4	1.5	1	1	2	1024	1024	1024
B6	free/(9)	1×10^4	piecewise (S8)	0.6	1.5	1	1	2	1024	1024	1024
C1	free/(9)	1×10^4	piecewise (S8)	0.4	0.0	5	-	2	5120	1	1024
C2≡B2	free/(9)	1×10^4	piecewise (S8)	0.4	1.5	5	-	2	5120	1	1024
C3	free/(9)	1×10^4	piecewise (S8)	0.4	3.0	5	-	2	5120	1	1024
C4	free/(9)	1×10^4	piecewise (S8)	0.4	0.0	1	1	2	1024	1024	1024
C5≡B5	free/(9)	1×10^4	piecewise (S8)	0.4	1.5	1	1	2	1024	1024	1024
C6	free/(9)	1×10^4	piecewise (S8)	0.4	3.0	1	1	2	1024	1024	1024

Sim.	interface/b.c.	Ra	$\rho(C)$	α	β	L_x	L_y	L_z	N_x	N_y	N_z
D1	free/(9)	1×10^4	piecewise (S8)	0.25	0.0	5	-	2	5120	1	1024
D2	free/(9)	1×10^4	piecewise (S8)	0.25	1.5	5	-	2	5120	1	1024
D3	free/(9)	1×10^4	piecewise (S8)	0.25	3.0	5	-	2	5120	1	1024
D4	free/(9)	1×10^4	piecewise (S8)	0.5	0.0	5	-	2	5120	1	1024
D5	free/(9)	1×10^4	piecewise (S8)	0.5	1.5	5	-	2	5120	1	1024
D6	free/(9)	1×10^4	piecewise (S8)	0.5	3.0	5	-	2	5120	1	1024
D7	free/(9)	1×10^4	piecewise (S8)	0.25	0.0	1	1	2	1024	1024	1024
D8	free/(9)	1×10^4	piecewise (S8)	0.25	1.5	1	1	2	1024	1024	1024
D9	free/(9)	1×10^4	piecewise (S8)	0.25	3.0	1	1	2	1024	1024	1024
D10	free/(9)	1×10^4	piecewise (S8)	0.5	0.0	1	1	2	1024	1024	1024
D11	free/(9)	1×10^4	piecewise (S8)	0.5	1.5	1	1	2	1024	1024	1024
D12	free/(9)	1×10^4	piecewise (S8)	0.5	3.0	1	1	2	1024	1024	1024
E1	free/(9)	1×10^4	piecewise (S8)	0.1	0.0	5	-	2	5120	1	1024
E2	free/(9)	1×10^4	piecewise (S8)	0.6	0.0	5	-	2	5120	1	1024
E3	free/(9)	1×10^4	piecewise (S8)	0.1	3.0	5	-	2	5120	1	1024
E4	free/(9)	1×10^4	piecewise (S8)	0.6	3.0	5	-	2	5120	1	1024
E5	free/(9)	1×10^4	piecewise (S8)	0.1	0.0	1	1	2	1024	1024	1024
E6	free/(9)	1×10^4	piecewise (S8)	0.6	0.0	1	1	2	1024	1024	1024
E7	free/(9)	1×10^4	piecewise (S8)	0.1	3.0	1	1	2	1024	1024	1024
E8	free/(9)	1×10^4	piecewise (S8)	0.6	3.0	1	1	2	1024	1024	1024